40. *On the Existence of Global Solutions of Mixed Problem for Non-linear Boltzmann Equation*

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1. Introduction and summary. We study a mixed initial-boundary value problem of non-linear Boltzmann equation. The boundary condition considered here is the periodicity condition, to which the perfectly reflective boundary condition for the case of a rectangular domain can be reduced, [1]. Our hypotheses on collision operators are those for the so-called cut-off hard potentials, [2]. The solutions for the mixed problem have been known to exist only locally in time, [1]. Our aim is to show their global existence.

We denote by $f=f(t,x,\xi)$ the density distribution of gas particles at time $t\geqslant 0$ with respect to the position $x=(x_1,x_2,x_3)\in \boldsymbol{R}^3$ and velocity $\xi=(\xi_1,\xi_2,\xi_3)\in \boldsymbol{R}^3$. Our mixed problem is

(1.1 a) $$\frac{\partial f}{\partial t}=-\sum_{i=1}^{3}\xi_i\frac{\partial f}{\partial x_i}+Q[f,f],$$

(1.1 b) $$f \text{ is periodic in } x,$$

(1.1 c) $$f|_{t=0}=f_0\geqslant 0,$$

(1.2) $$Q[f,g]=\frac{1}{2}\iint_{R^3\times S^2} q(|\xi-\xi'|,\theta)\{f(\eta)g(\eta')+f(\eta')g(\eta)-f(\xi)g(\xi') -f(\xi')g(\xi)\}d\xi' d\omega.$$

Here $f(\eta)=f(t,x,\eta)$, etc., while $\omega\in S^2$, $\cos\theta=(\omega,\xi-\xi')/|\xi-\xi'|$, $\eta=\xi+(\omega,\xi-\xi')\omega$ and $\eta'=\xi'-(\omega,\xi-\xi')\omega$. The assumption of the cut-off hard potential means, [2], that there exist constants $q_0,q_1>0$, $0\leqslant\delta<1$ and for any $v>0$,

(1.3) $$0\leqslant q(v,\theta)\leqslant q_0|\cos\theta|(v+v^{-\delta}),\qquad \int_0^{\pi} q(v,\theta)\sin\theta d\theta\geqslant q_1 v(1+v)^{-1}.$$

Let $\Omega\subset\boldsymbol{R}^3$ be a fundamental rectangular domain of the periodicity condition. Let $g(\xi)=e^{-|\xi|^2}$ be a Maxwellian (Gaussian) distribution. With suitable changes of variables x, ξ and t, we may assume that, with $\{h_i(\xi)\}_{i=1}^5=\{1,\xi_1,\xi_2,\xi_3,|\xi|^2\}$,

(1.4) $$\iint_{\Omega\times R^3} h_i(\xi)f_0(x,\xi)dxd\xi=\iint_{\Omega\times R^3} h_i(\xi)g(\xi)dxd\xi,\qquad 1\leq i\leq 5.$$

Put $\psi_i=h_i g^{\frac{1}{2}}$. Define the (formal) operators L and Γ as

(1.5) $$Lu=g^{-\frac{1}{2}}Q[g^{\frac{1}{2}},g^{\frac{1}{2}}u],\qquad \Gamma[u,v]=g^{-\frac{1}{2}}Q[g^{\frac{1}{2}}u,g^{\frac{1}{2}}v].$$

Under the assumption (1.3), L takes the form, [2],