38. On Some New Invariants of Polarized Manifolds

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(Comm. by Kunihiko KODAIRA, M. J. A., March 12, 1974)

In this note we shall announce a couple of theorems on some invariants of polarized manifolds, which will be useful in our study of their structures (see [2]). Details will be published elsewhere.

First we review some results in [1], in which we defined the following three invariants of a polarized manifold, i.e., a pair (M, F) of a compact complex manifold M and an ample line bundle F on M:

$d(M, F)=F^n=(c_1(F))^n[M]$, where $n=\dim M$,

$\Delta(M, F)=n+d(M, F)-\dim H^0(M, \mathcal{O}_M(F))$,

$2g(M, F)-2=(K_M+(n-1)F)F^{n-1}$, where K_M is the canonical bundle of M.

We call $\Delta(M, F)$ the *Δ-genus* of (M, F). Note that if D is a non-singular member of $|F|$, then $d(D, F_D)=d(M, F)$, $g(D, F_D)=g(M, F)$ and $\Delta(D, F_D) \leqq \Delta(M, F)$, where F_D is the restriction of F to D. Moreover $\Delta(D, F_D) = \Delta(M, F)$ if $H^1(M, \mathcal{O}_M)=0$ or $H^1(D, \mathcal{O}_D)=0$. In [1] we established the inequality $\dim Bs|F| < \Delta(M, F)$, where $Bs|F|$ is the set of the base points of $|F|$. This assured us of the existence of a non-singular member of $|F|$ if $\Delta(M, F)=0$, and enabled us to classify such polarized manifolds.

Now we give a sufficient condition for the existence of a non-singular member of $|F|$ and state some of its applications.

Theorem I. *Let (M, F) be a polarized manifold with $g(M, F) \geqq \Delta(M, F)$ and $\dim Bs|F| \leqq 0$. Then $|F|$ has a non-singular member if $d(M, F) \geqq 2\Delta(M, F)-1$.*

Corollary I–1. *Suppose, in addition, that $d(M, F) \geqq 2\Delta(M, F)$. Then $Bs|F|=\emptyset$.*

Corollary I–2. *Suppose, in addition, that $d(M, F) \geqq 2\Delta(M, F)+1$. Then $g(M, F)=\Delta(M, F)$.*

Corollary I–3. *Under the same conditions as in Theorem I, let D be a non-singular member of $|F|$. Then $\Delta(D, F_D)=\Delta(M, F)$.*

Using these results, we can prove the following theorem by induction on $\dim M$.

Theorem II. *Let (M, F) be a polarized manifold with $g(M, F) \geqq \Delta(M, F)$ and $\dim Bs|F| \leqq 0$. Then F is very ample if $d(M, F) \geqq 2\Delta(M, F)+1$.*

Remark. When M is a curve, the conditions $g(M, F) \geqq \Delta(M, F)$ and $\dim Bs|F| \leqq 0$ are always satisfied if $d(M, F) \geqq 2\Delta(M, F)-1$. Hence