69. Closeness Spaces and Convergence Spaces

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The purpose of this note is to show that every convergence structure ("Limitierung" of Fischer [2]) can be described by a family, called a *closeness*, of closure-like operations.

After stating several elementary properties of operations on the power set of a set, we shall introduce new notions "closeness" and "closeness space". Then some fundamental relations between closenesses and convergence structures will be established.

In what follows, the power set of a set X will be denoted by $\wp(X)$, and the value of a mapping $\alpha:\wp(X)\to\wp(X)$ at $A\in\wp(X)$ by A^α. The complement of $A\in\wp(X)$ in X will be written A^c. For each $x\in X$, $\dot{x}$ denotes the filter on X consisting of all $A\in\wp(X)$ with $x\in A$.

1. Throughout this section X denotes an arbitrary set. Let α be a mapping of $\wp(X)$ into itself. For each $x\in X$, we denote by $\Phi_\alpha(x)$ the set of all $A\in\wp(X)$ such that $x\notin A^{c\alpha}$. Evidently Φ_α is a mapping of X into $\wp\wp(X)=\wp(\wp(X))$.

The following four lemmas may be easily verified, and we omit the proofs.

Lemma 1. *Let α be a mapping of $\wp(X)$ into itself, and let $x\in X$. Then the following statements hold:*

(1) *$\Phi_\alpha(x)\neq\emptyset$ if and only if x does not belong to $\cap\{A^\alpha|A\in\wp(X)\}$.*

(2) *$\emptyset\notin\Phi_\alpha(x)$ if and only if $x\in X^\alpha$.*

Lemma 2. *Let α be a monotone mapping*) of $\wp(X)$ into itself. Then $x\in\{x\}^\alpha$ for every $x\in X$ if and only if $A\subset A^\alpha$ for every $A\in\wp(X)$.*

Lemma 3. *Let α be a monotone mapping of $\wp(X)$ into itself, and let $A\in\wp(X)$. Then $x\in A^\alpha$ if and only if $S\cap A\neq\emptyset$ for every $S\in\Phi_\alpha(x)$.*

Lemma 4. *Let α,β be two monotone mappings of $\wp(X)$ into itself. Then $\Phi_\alpha(x)\subset\Phi_\beta(x)$ for every $x\in X$ if and only if $A^\beta\subset A^\alpha$ for every $A\in\wp(X)$.*

Let Ψ be a mapping of X into $\wp\wp(X)$. For each $A\in\wp(X)$, we denote by $A^{\kappa(\Psi)}$ the set of all $x\in X$ for which we have $S\cap A\neq\emptyset$ for every $S\in\Psi(x)$. Obviously $\kappa(\Psi)$ is a monotone mapping of $\wp(X)$ into itself. Conversely, as an immediate consequence of Lemma 3, we have the following

*) A mapping α of $\wp(X)$ into itself is called *monotone* if $A\subset B$ implies $A^\alpha\subset B^\alpha$ for every $A,B\in\wp(X)$.