67. *Kneser's Property of Solution Families of Non-linear Volterra Integral Equations*

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Consider a system of nonlinear integral equations of Volterra-type

$$(\mathrm{P}) \qquad x(t)=f(t)+\int_0^t g(t,s,x(s))ds.$$

Recently R. K. Miller and G. R. Sell [1] proved some fundamental theorems of (P) under fairly general assumptions on $f(t)$ and $g(t,s,x)$ similar to the Carathèodory-type. They showed that the cross-section

$$F(t)=\{y : y=x(t), \text{ where } x \text{ is some solution of (P)}\}$$

is compact in R^n for all $t\in[0,\alpha_M)$, where α_M is either ∞ or a finite number such that there is a solution $x(t)$ of (P) for which $\limsup_{t\uparrow\alpha_M}|x(t)|=\infty$. This appears to be a generalization of H. Kneser's theorem to integral equations.

For the case where $g(t,s,x)$ is a bounded continuous function of (t,s,x) on $\{0\leqq s\leqq t\leqq\alpha\}\times R^n$, Sato [3] has shown that $F(t)$ is a continuum, i.e., a compact and connected set for all $t\in[0,\alpha]$. One of the present authors later proved in [4] that the family of all solution-curves is a continuum even in $\mathcal{C}[0,\alpha]$.

We think that it is interesting to know whether $F(t)$ is a continuum or not for all $t\in[0,\alpha_M)$ under the weaker assumptions of Miller and Sell. The purpose of this note is to give an answer in the affirmative for this problem. Moreover, we can demonstrate that the family of solutions of (P) is also a continuum in the Frèchet space $\mathcal{C}[0,\alpha_M)$.[1)]

Since the method we employed in this paper mainly depends on Carathèodory iterates, there is no need in our proof to use the approximate functions g_n to g satisfying the Lipschitz condition which was employed in [3] and [4].

We assume the hypotheses (H1)–(H5) on $f(t)$ and $g(t,s,x)$ used in our previous note [5]. We shall show the following main theorem.

Theorem. *Let the functions f and g satisfy* (H1)–(H5), *then there exists a number $\alpha_M>0$ such that for each $t\in[0,\alpha_M)$ the set $F(t)$ is compact and connected as a subset of R^n. Moreover the number α_M is*

1) After completing this manuscript, we found that W. G. Kelly (Proc. Amer. Math. Soc., **40**, 1973) proved a local Kneser property, that is, the set $\{x(t)\in\mathcal{C}[0,d];\ x(t)$ is a solution of (P) on $[0,d]\}$ is compact and connected in the space $\mathcal{C}[0,d]$ for any $d<\alpha_M$.