66. *Quantitative Properties of Analytic Varieties*

Complex Analytic De Rham Cohomology. II

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This note is a continuation of [3]. The purpose of this note is to outline our recent results on certain quantitative properties of *real* analytic varieties. Details will appear elsewhere. The results will provide a *topological* key to the complex analytic De Rham cohomology theory. In what follows we are basically concerned with asymptotic and division properties of C^∞-differentiable differential forms with respect to given real analytic varieties. In this note we always mean by a variety a real analytic variety and we abbreviate the word C^∞-differentiable as C^∞. The symbols $L, N(Q, V)$, etc., have the same meanings as in [3]. For a fixed system of coordinates $(x)=(x_1, \cdots, x_n)$ of $\boldsymbol{R}^n$, $D_K=\partial^{|K|}/\partial x^K$, where $K=(k_1, \cdots, k_n)$, $x^K=x_1^{k_1}\cdots x_n^{k_n}$. Let $\mathcal{D}$ be a domain in R^n and W a closed subset of $\mathcal{D}$. A C^∞-function f in $\mathcal{D}-W$ is said to be of *polynomial growth with respect to* W if, for each K, there exists a couple a_K such that $|D_K f(Q)| \leqq a_{K1}\cdot d(Q, W)^{-a_{K2}}$. A C^∞-form $\varphi=\sum_J \varphi_J dx^J$ in $\mathcal{D}-W$ will be said to be of *polynomial growth with respect to* W if each coefficient φ_J is of polynomial growth.

Let (U, V, P) be a datum composed of a domain U in R^n, a variety V in U and a point P in V. This datum will be fixed throughout this note. First we state our results in terms of varieties in question and of coordinates (x).

n.1. *C^∞-thickenings and their quantitative properties.* Consider a proper subvariety $V' \ni P$ of V in addition to the datum (U, V, P). For a couple σ, let $N_\sigma(V : V')$ denote the neighbourhood of $V-V'$ defined by $N_\sigma(V : V')=\bigcup_{Q\in V-V'} N_\sigma(Q : V')$. A neighbourhood N of $V-V'$ is called a *C^∞-thickening* of $V-V'$, if $H^*(V-V' : \boldsymbol{R})\cong H^*(N : \boldsymbol{R})$. Let $\{N_j : j\in \boldsymbol{Z}\}$ be a direct system of C^∞-thickenings with respect to the inclusion relation satisfying the following conditions:

(1) For any N_j there exists a couple σ_j such that $N_j \supset N_{\sigma_j}(V : V')$.

(2) For an arbitrary σ, $N_j \subset N_\sigma(V : V')$ for a sufficiently large j.

For a neighbourhood N of $V-V'$, $\Omega(N)$ denotes the ring of C^∞-differential forms in N. Moreover, we understand by $\Omega(N : V')$ the subring of $\Omega(N)$ composed of those forms which are of polynomial growth with respect to V'. Given a direct system $\{N_j : j\in \boldsymbol{Z}\}$ of C^∞-