65. *On an Invariant of Veronesean Rings*

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§ 1. **Main result.** Let K be a field and $t_1, \cdots, t_n$ indeterminates. Let m be a positive integer. In this paper we consider the ring $R_{n,m}$ generated, over K, by all the monomials $t_1^{p_1} \cdots t_n^{p_n}$ such that $\sum_{i=1}^{n} p_i = m$. Let $S_{n,m}$ be the localization of $R_{n,m}$ at the maximal ideal generated by all $t_1^{p_1} \cdots t_n^{p_n}$ in $R_{n,m}$. In [2] Gröbner showed that the local ring $S_{n,m}$ is a Macaulay ring of dimension n. In this paper this ring is called a *Veronesean* local ring.

In general, it is well known that in a Macaulay local ring the number of the irreducible components of an ideal generated by a system of parameters is an invariant of the ring. This invariant is called the *type* of the ring (cf. [4]). A Macaulay local ring is a Gorenstein ring if and only if the ring has type one.

The aim of this paper is to prove the following theorem.

Theorem. *Let $S_{n,m}$ be a Veronesean local ring. Then*

$$\text{type}\, S_{n,m} = 1 \qquad \text{if } n \equiv 0 \ (\text{mod. } m)$$

and

$$\text{type}\, S_{n,m} = \binom{n+m-r-1}{n-1} \quad \text{if } n \equiv r \ (\text{mod. } m) \quad 0 < r < m.$$

As a direct consequence of the theorem, we have the following

Corollary. *A Veronesean local ring $S_{n,m}$ is a Gorenstein ring if and only if $n=1$ or $n \equiv 0$ (mod. m).*

§ 2. **Proof of theorem.** For a non-negative integer s, we denote by $\mathrm{P}(s)$ the set of ordered n-tuples $(p) = (p_1, \cdots, p_n)$ of non-negative integers p_i such that $\sum_{i=1}^{n} p_i = sm$. We also denote by $t^{(p)}$ the monomial $t_1^{p_1} \cdots t_n^{p_n}$. With the same notation as in § 1, the ring $R_{n,m} = K[t^{(p)} \mid (p) \in P(1)]$. Let $\mathfrak{m}$ be the maximal ideal generated by all $t^{(p)}, (p) \in \mathrm{P}(1)$, and $\mathfrak{q}$ the ideal generated by $t_1^m, \cdots, t_n^m$. Then $\mathfrak{q}$ is an $\mathfrak{m}$-primary ideal. Since the localization $S_{n,m}$ of $R_{n,m}$ at $\mathfrak{m}$ is a Macaulay local ring of dimension n and since $\{t_1^m, \cdots, t_n^m\}$ is a maximal regular sequence of $S_{n,m}$ (cf. [2]), the type of $S_{n,m}$ is given by the dimension of the K-vector space $(\mathfrak{q} : \mathfrak{m})/\mathfrak{q}$ (cf. [4]).

Before proving some lemmas we give preliminary remarks: A monomial $t^{(p)}$ is in $R_{n,m}$ if and only if (p) is in $\mathrm{P}(s)$ for some s. If (p)