64. Wave Equation with Wentzell's Boundary Condition and a Related Semigroup on the Boundary. II

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1. In part I of this paper [1], we defined a closure $\bar{A}_L$ of A with respect to Wentzell's *boundary condition*

$$Lu(x)=0, \qquad x \in \partial D,$$

and solved the *wave equation*

$$(1) \qquad \frac{\partial^2}{\partial t^2}u=\bar{A}_L u, \quad u(t,\cdot)\to f, \quad \frac{\partial}{\partial t}u(t,\cdot)\to g, \qquad \text{as } t\to 0,$$

by solving the equations of type

$$(2) \qquad \alpha u-\bar{A}_L u=v, \qquad \text{for } v\in \mathcal{H},$$

and using the scheme in **2** of [1].

Here, we consider L as an operator which maps a function u on $\bar{D}$ to a function Lu on ∂D, and define a closure $\bar{L}_A$ of L with respect to the *domain condition*

$$(3) \qquad Au(x)=0, \qquad x\in D,$$

just as we defined $\bar{A}_L$. Since each function in $\mathcal{D}(\bar{L}_A)$ can be proved to satisfy (3), it is written as $H\varphi(x)=\int_{\partial D} H(x,dy)\varphi(y)$ by the boundary value φ and the harmonic measure $H(x,\cdot)$ with respect to the domain D and point x.[1] Thus, we define $\overline{LH}$ by $\overline{LH}\varphi=\bar{L}_A H\varphi$ on $\{\varphi\in\mathcal{H}_\partial \mid H\varphi \in D(\bar{L}_A)\}$, where $\mathcal{H}_\partial$ is the Hilbert space of all measurable functions on ∂D such that $\|\varphi\|_\partial=\langle\varphi,\varphi\rangle^{\frac{1}{2}}<\infty$. Then, we can solve

$$(4) \qquad \frac{\partial^2}{\partial t^2}\varphi=\overline{LH}\varphi, \quad \varphi(t,\cdot)\to\psi, \quad \frac{\partial}{\partial t}\varphi(t,\cdot)\to\eta, \qquad \text{as } t\to 0,$$

by using the scheme in **2** of [1] and solving the equations of type

$$(5) \qquad \lambda[u]_\partial-\bar{L}_A u=\varphi, \qquad \text{for } \varphi\in\mathcal{H}_\partial,$$

where $[u]_\partial$ is the restriction of u to the boundary ∂D.

It is expected that the mapping L and the equation (4) have some intuitive meanings, closely related with (1). Some comments on this point will be added in comparison with equation

$$(6) \qquad \frac{\partial}{\partial t}\varphi=\overline{LH}\varphi, \quad \varphi(t,\cdot)\to\psi, \qquad \text{as } t\to 0,$$

1) The harmonic measure corresponds to $A=\Delta$. For a general A, a measure with similar properties exists, and it is sometimes called the *hitting measure*. In fact, this is the probability distribution of the first hit to the boundary of the diffusion particle corresponding to A and started at point x.