56. A Remark of a Neukirch's Conjecture

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Introduction. Let Q be the rational number field, $\bar{Q}$ the algebraic closure of Q and G_Q the Galois group of $\bar{Q}$ over Q with Krull topology. In [4] Neukirch gave a conjecture to the effect that any topological automorphism of G_Q is inner. In this paper we shall show the following affirmative datum:

Theorem. *Let α be a topological automorphism of G_Q. Then for any element τ in G_Q, there exists an element σ_τ in G_Q such that $\alpha(\tau)=\sigma_\tau^{-1}\tau\sigma_\tau$.*

Some properties of decomposition groups of non-archimedean valuations, which we shall use to get the above theorem, also shall be stated with a result that the center of G_Q is trivial.

§ 1. The center of G_k. Let Q be the rational number field and $\bar{Q}$ the algebraic closure of Q. For any subfield K of $\bar{Q}$, let G_K be the topological Galois group of $\bar{Q}$ over K. In this paper field means a subfield of $\bar{Q}$.

Definition 1. Let K be a subfield of $\bar{Q}$ and v a non-archimedean valuation of K. K is said to be henselian with respect to v if an extension of v to $\bar{Q}$ is unique.

Lemma 1 (cf. [1]). *For a proper subfield K of $\bar{Q}$, let v_1 and v_2 be non-archimedean valuations of K. If K is henselian with respect to v_1 and v_2, then v_1 and v_2 are equivalent as valuation.*

Let k be a subfield of $\bar{Q}$ and $\bar{v}$ a non-archimedean valuation of $\bar{Q}$. We denote by $D_k(\bar{v})$ the decomposition group of $\bar{v}$ in G_k and by $N_k(D_k(\bar{v}))$ the normalizer of $D_k(\bar{v})$ in G_k. Since $D_k(\bar{v})$ is a closed subgroup of G_k, there exists the subfield K of $\bar{Q}$ such that $G_K=D_k(\bar{v})$. Then K is henselian with respect to the restriction $\bar{v}|_K$ of $\bar{v}$ to K. We denote by x^σ the image of an element x in $\bar{Q}$ by an automorphism σ in G_Q and by $\bar{v}^\sigma$ the valuation of $\bar{Q}$ such that $\bar{v}^\sigma(x)=\bar{v}(x^\sigma)$ for any element x in $\bar{Q}$. Then we have

$$(1) \qquad D_k(\bar{v}^\sigma)=\sigma D_k(\bar{v})\sigma^{-1}$$

for any element σ in G_k.

Lemma 2. *If k is a finite extension of Q, then we have $D_k(\bar{v})=N_k(D_k(\bar{v}))$ for any non-archimedean valuation $\bar{v}$ of $\bar{Q}$.*

Proof. It is clear that $D_k(\bar{v})$ is contained in $N_k(D_k(\bar{v}))$. So it is sufficient to show that $\bar{v}^\sigma=\bar{v}$ for any element σ in $N_k(D_k(\bar{v}))$. Let σ be