# 81. *Harmonic Analysis on Some Types of Semisimple Lie Groups*

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**1. Introduction.** Let $G$ be a semisimple Lie group and let $L^2(G)$ denote the space of square integrable functions on $G$ with respect to the Haar measure. The Fourier transform $\mathcal{F}$ can be regarded as an isometry of $L^2(G)$ onto a Hilbert space $L^2(\hat{G})$, which is defined with the help of some types irreducible unitary representations of $G$.

In his paper [5], Harish-Chandra introduces the Schwartz space $\mathcal{C}(G)$, of functions on $G$. It is analogous to the Schwartz space $\mathcal{S}(\boldsymbol{R}^n)$ ([6]), of rapidly decreasing functions on a euclidean space $\boldsymbol{R}^n$, and is contained densely in $L^2(G)$.

It is an interesting problem to characterize the image $\mathcal{C}(\hat{G})$ of $\mathcal{C}(G)$ in $L^2(\hat{G})$ by the Fourier transform $\mathcal{F}$. In the case that the real rank of $G$ equals one, J. G. Arthur [1] solves this problem. Moreover, these problems for certain subspaces of $\mathcal{C}(G)$ are studied in the papers [3], [4] and [2].

The purpose of this paper is to give a characterization of the Fourier image $\mathcal{C}(\hat{G})$ for non-compact real semisimple Lie groups $G$ with only one conjugacy class of Cartan subgroups.

The difficult part of this theorem is to prove surjectivity. For this, we must study in detail the asymptotic behaviour of the Eisenstein integrals, in particular, not only the constant terms but the asymptotic behaviour of them along the walls of Weyl chambers.

Detailed proofs will appear elsewhere.

**2. Notation and preliminaries.** Let $G$ be a connected non-compact real semisimple Lie group with finite center. Let $\mathfrak{g}$ be the Lie algebra of $G$. Throughout this paper, we assume that $G$ has only one conjugacy class of Cartan subgroups and $\mathfrak{g}$ does not have any complex structure. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be a fixed Cartan decomposition with Cartan involution $\theta$, $\mathfrak{a}_\mathfrak{p}$ a maximal abelian subspace of $\mathfrak{p}$ and $\mathfrak{a}_\mathfrak{p}^*$ its dual space respectively. Let $\mathfrak{a}$ be a Cartan subalgebra of $\mathfrak{g}$ which contains $\mathfrak{a}_\mathfrak{p}$ and put $\mathfrak{a}_\mathfrak{k}=\mathfrak{a}\cap\mathfrak{k}$. Let $\mathfrak{g}^c$ and $\mathfrak{a}^c$ be the complexifications of $\mathfrak{g}$ and $\mathfrak{a}$ respectively, and $\Delta$ denote the set of non-zero roots of $\mathfrak{g}^c$ with respect to $\mathfrak{a}^c$. We introduce a linear order in $\Delta$ which is compatible with respect to $\mathfrak{a}_\mathfrak{p}$. Let $\Delta_+$ and $P_+$ denote the set of positive roots and those which do not vanish on $\mathfrak{a}_\mathfrak{p}$. Let $P_-$ denote the complement of $P_+$ in $\Delta_+$. Define