79. *Fourier Transform of Banach Algebra Valued Functions on Group*

By Reri TAKAMATSU
Department of Mathematics, Sophia University

(Comm. by Kôsaku YOSIDA, M. J. A., June 11, 1974)

1. Introduction and preliminaries. Let G be a locally compact group with unit element e, and A be a complex Banach algebra with unit element 1.

Through this paper, we let Haar measure of non abelian group be left invariant, and we let $\int dx, \int dy, \cdots$, denote integration with respect to Haar measure and $m(E)$ the Haar measure of a set E.

We denote the Fourier transform $\hat{f}$ of $f \in L^1(G)$, when G is abelian, by

$$\hat{f}(\gamma)=\int_G f(x)(-x,\gamma)dx \qquad (\gamma \in \Gamma\,;\text{ the dual group of } G).$$

A well known theorem states that a functional h defined on $L^1(G)$ is a non-zero complex homomorphism if and only if

$$h(f)=\hat{f}(\gamma) \quad (f \in L^1(G)) \quad \text{for some } \gamma \in \Gamma.$$

In this paper, we give an analogue of this theorem by replacing the functions $f \in L^1(G)$ with A-valued functions on G. This is also a preliminary step to get formally a unified view about the group algebra and the representation of groups by linear transformations on a vector space, which form a Banach algebra.[1)]

Let $C_0(G\to A)$ denote the set of all A-valued continuous functions on G with compact support, and $L^1(G\to A)$ denote the completion of $C_0(G\to A)$ with respect to the norm $|||\cdot|||$, defined by

$$|||f|||=\int_G \|f(x)\|\,dx.$$

We say an A-valued function f on G is a measurable step function on G if $f(x)$ is of the form

$$f(x)=\sum_{\nu=1}^{n} a_\nu \chi_{E_\nu}(x),$$

where $a_\nu \in A$ and E_ν are measurable sets (with respect to Haar measure) with compact closure, and χ_{E_ν} are characteristic functions of E_ν.

The proofs of Proposition 1 and 2 will be given easily.

Proposition 1. *The set of all measurable step functions is dense in* $L^1(G\to A)$.

1) L. Loomis §31 and §32.