78. *On K. Yosida's Class* (A) *of Meromorphic Functions*

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1. Introduction. The class (A) in K. Yosida's sense [5] consists of all functions f meromorphic in the plane $C: |z|<+\infty$ such that the family $\{f_\alpha\}, \alpha \in C$, of functions $f_\alpha(z)=f(z+\alpha), z \in C$, is normal in the sense of P. Montel in C. We set $k(f)=\sup_{z\in C} f^*(z)$ for $f \in (A)$, where $f^*(z)=|f'(z)|/(1+|f(z)|^2)$; we know that $k(f)<+\infty$ [5, Theorem 1]. Plainly, $k(f)>0$ if and only if f is non-constant. Given a function f meromorphic in C and a point $z \in C$, let $u(z)=u(z,f)$ be the supremum of $r>0$ such that f is univalent in the disk $D(z,r)=\{w \in C; |w-z|<r\}$; if such an r does not exist, we set $u(z)=u(z,f)=0$. Then $u(z)=0$ if and only if $f^*(z)=0$. Except for the case that f is linear, $u(z)<+\infty$ at each $z \in C$. Furthermore, a non-linear f is univalent in $D(z,u(z))$ and the function u is continuous in C (Lemma). Here and elsewhere a meromorphic function f is called non-linear if f is non-constant and not linear. We begin with

Theorem 1. *Given a non-linear f of class (A), we have at each $z \in C$,*

$$f^*(z) \leqq (32/\pi^2)k(f)^2 u(z,f). \tag{1}$$

Of course, the estimate (1) has the good meaning if $u(z,f) <\pi^2/\{32k(f)\}$. As an application of Theorem 1 we know that $u(z_n,f) \to 0$ implies $f^*(z_n)\to 0$ for each sequence of points $\{z_n\}\subset C$ converging to a point of C or else to the point at infinity. However, the converse is not valid; the exponential function $E(z)=e^z$ belongs to (A) with $u(z,E)=\pi$ at each $z \in C$ but $E^*(n)\to 0$ as $n\to +\infty$, n being positive integers.

Our next result concerns the derived function.

Theorem 2. *Given a non-linear f of class (A), we have at each $z \in C$,*

$$f'^*(z) \leqq 2[\min\{k(f)^{-1}, u(z,f)\}]^{-1}+1, \tag{2}$$

where $f'^*(z)=|f''(z)|/(1+|f'(z)|^2)$.

The function $E \in (A)$ has the property that $E' \in (A)$, which suggests the following application of Theorem 2. We have $f' \in (A)$ if $f \in (A)$ and if $\inf_{|z|>R} u(z,f)>0$ for a certain constant $R>0$. Indeed, f'^* is bounded in $|z|>R$ by (2), while f'^* is bounded in $|z|\leqq 2R$ because f'^* is continuous in C, whence f'^* is bounded in C. Therefore $f' \in (A)$ by