77. *Oscillation Theorems for Second Order Differential Equations with Retarded Argument*

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Introduction. In this paper we are concerned with the oscillatory behavior of solutions of the differential equation with retarded argument

(A) $$(r(t)x'(t))'+a(t)f(x(g(t)))=0,$$

where the following conditions are always assumed to hold:

(a) $r(t)\in C^1(0,\infty)$, $r(t)>0$;

(b) $a(t)\in C(0,\infty)$, $a(t)\geqq 0$;

(c) $g(t)\in C^1(0,\infty)$, $g(t)\leqq t$, $g'(t)\geqq 0$, $\lim_{t\to\infty} g(t)=\infty$;

(d) $f(y)\in C(-\infty,\infty)\cap C^1(-\infty,0)\cap C^1(0,\infty)$, $yf(y)>0$, $f'(y)\geqq 0$ for $y\neq 0$.

We consider only those solutions of (A) which are defined and nontrivial for all sufficiently large t. Such a solution is called oscillatory if it has arbitrarily large zeros; otherwise, it is called nonoscillatory.

Our purpose here is to present criteria (sufficient conditions) for all solutions of (A) to be oscillatory not only for the case $\int^\infty \frac{dt}{r(t)}=\infty$ but also for the case $\int^\infty \frac{dt}{r(t)}<\infty$. Our theorems can be applied to produce oscillation criteria for the damped equation

(B) $$x''(t)+p(t)x'(t)+q(t)f(x(g(t)))=0.$$

1. We begin with the case $\int^\infty \frac{dt}{r(t)}=\infty$. In this case the following theorem holds.

Theorem 1. *Assume there exist two positive functions $\rho(t)\in C^2(0,\infty)$ and $\phi(y)\in C^1(0,\infty)$ with the following properties:*

$$\rho'(t)\geqq 0,\quad (r(t)\rho'(t))'\leqq 0,\quad \phi'(y)\geqq 0,$$

$$\int_{\pm\delta}^{\pm\infty}\frac{dy}{f(y)\phi(y)}<\infty \qquad \textit{for some } \delta>0,$$

$$\int^\infty \frac{\rho(g(t))a(t)}{\phi(R_T(g(t)))}dt=\infty \qquad \textit{for any } T>0,$$

where $R_T(t)=\int_T^t \frac{ds}{r(s)}$. *Then all solutions of* (A) *are oscillatory.*

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