74. A Generalization of Bieberbach's Example

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1. Bieberbach constructed an example of a biholomorphic mapping of C^2 onto a proper open subset of C^2 ([1], see also [3]). His construction depends on the following fact. Let $g: z \to g(z)$ be a complex analytic automorphism of C^2 of which the origin 0 is a fixed point $g(0)=0$. The automorphism g induces a linear transformation of the tangent space $T_0(C^2)$ $(\simeq C^2)$ of C^2 at 0. Assume that the eigenvalues α_1, α_2 of the linear transformation satisfy $1>|\alpha_1|\geq|\alpha_2|$. Then the set

$$U=\left\{z \in C^2 : \lim_{\nu \to +\infty} g^{\nu}(z)=0\right\}$$

is complex analytically isomorphic to C^2. The purpose of this paper is to generalize the above fact. Namely we shall prove

Theorem. *Let X be a complex space of dimension m. Assume that there exists a complex analytic automorphism g and a point $0 \in X$ such that $g(0)=0$ and $g^{\nu}(z) \to 0$ $(\nu \to +\infty)$ for any point $z \in X$. Then X is complex analytically isomorphic to an affine variety. If, moreover, X is non-singular at 0, then $X \simeq C^m$.*

In [2], it is shown that the latter statement holds and that, if X is singular, X can be embedded into C^n as a closed subvariety which is invariant under a contracting complex analytic automorphism $\tilde{g}$ of C^n such that $\tilde{g}(0)=0$ and $\tilde{g}_{|X}=g$, where 0 denotes the origin of C^n. Let $(z_1, \cdots, z_n)$ be a standard system of coordinates of C^n. We may assume that $\tilde{g}$ has the following form;

$$
(1)\quad
\begin{aligned}
&z_1'=\alpha_1 z_1\\
&z_2'=z_1+\alpha_1 z_2\\
&\vdots\\
&z_{r_1}'=z_{r_1-1}+\alpha_1 z_{r_1}\\
&z_{r_1+1}'=\alpha_2 z_{r_1+1}+P_{r_1+1}(z_1, \cdots, z_{r_1})\\
&\vdots\\
&z_{r_1+r_2}'=z_{r_1+r_2-1}+\alpha_2 z_{r_1+r_2}+P_{r_1+r_2}(z_1, \cdots, z_{r_1})\\
&z_{r_1+r_2+1}'=\alpha_3 z_{r_1+r_2+1}+P_{r_1+r_2+1}(z_1, \cdots, z_{r_1}, z_{r_1+1}, \cdots, z_{r_1+r_2})\\
&\vdots\\
&z_n'=z_{n-1}+\alpha_\mu z_n+P_n(z_1, \cdots, z_{r_1+\cdots+r_{\mu-1}}),
\end{aligned}
$$

where $1>|\alpha_1|\geq|\alpha_2|\geq\cdots\geq|\alpha_\mu|>0$ and P_j $(r_1+\cdots+r_s<j\leq r_1+\cdots+r_{s+1})$ are finite sums of monomials $z_1^{m_1}\cdots z_{r_s}^{m_{r_s}}$ which satisfy $\alpha_{r_{s+1}}=\alpha_1^{m_1}\cdots\alpha_{r_s}^{m_{r_s}}$, $m_1+\cdots+m_{r_s}\geq 2$ and $m_l>0$ ([4], [5]).