109. *Shift Automorphism Groups of von Neumann Algebras*

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1. In the structure theory of von Neumann algebras of type III, Connes and Takesaki have treated a group G of automorphisms ($*$-preserving) of a von Neumann algebra $\mathcal{A}$ with the following property:

$$(*)\quad \begin{cases} \mathcal{A} \text{ admits a faithful semi-finite normal trace } \varphi \text{ such that} \\ \qquad\qquad \varphi\cdot g=\lambda_g\varphi \qquad\qquad (1) \\ \text{for every non trivial automorphism } g \text{ of } G \text{ and some scalar} \\ 0<\lambda_g\neq 1 \text{ depending on } g. \end{cases}$$

Especially, assume that G is a singly generated automorphism group of an abelian von Neumann algebra $\mathcal{A}$. It is proved that there exists a projection E of $\mathcal{A}$ such that

$$\{g(E)\,;\, g\in G\} \text{ is an orthogonal family} \qquad (2)$$

and

$$\sum_{g\in G} g(E)=1 \qquad (3)$$

if G satisfies the property $(*)$.

We have an interest in an automorphism group of a von Neumann algebra with such a projection.

Definition 1. Let G be an automorphism group of a von Neumann algebra $\mathcal{A}$. If there exists a projection E of $\mathcal{A}$ with (2) and (3), then G is called a *shift* and E is called a *shift projection* of G in $\mathcal{A}$. Especially, if E is a central projection, then G is called a *central shift.*

In this paper, we shall show, for a singly generated automorphism group, an elementary relation between the property $(*)$ and the notion of shift and prove the following theorem:

Theorem 2. *If G is a discrete central shift of automorphisms of a von Neumann algebra $\mathcal{A}$, then the crossed product of $\mathcal{A}$ by G is isomorphic to the tensor product $\mathcal{A}^G\otimes\mathcal{L}(L^2(G))$ of the fixed algebra $\mathcal{A}^G$ in $\mathcal{A}$ of G and the algebra $\mathcal{L}(L^2(G))$ of all bounded operators on $L^2(G)$.*

2. In order to construct the discrete crossed product of a von Neumann algebra $\mathcal{A}$ by an automorphism group G, freely acting automorphism groups play an important role.

An automorphism g of a von Neumann algebra $\mathcal{A}$ is called *freely acting* on $\mathcal{A}$
when

$$AB=g(B)A \qquad \text{for all } B \text{ in } \mathcal{A}$$

implies