107. *Initial-Boundary Value Problems of Some Non-Linear Evolution Equations in Orlicz-Sobolev Spaces*

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1. Introduction. Let Ω be a bounded domain in R^n with boundary $\partial\Omega$. Recently in his paper [1] T. Donaldson proved the existence of weak solutions (in some Orlicz-Sobolev spaces) of non-linear elliptic boundary value problems of which are given two examples

(1.1) $$\sum_{i,j\leqslant n} D_i(u\cdot\exp(D_ju)^2)+D_n(\exp\beta(D_nu)^2)=f,\ \beta>0$$

and

(1.2) $$\sum_{i,j\leqslant n} D_i(D_ju)^2\ln(D_ju)^2=f$$

both associated with the boundary condition $u|_{\partial\Omega}=0$.

Originally Leray and Lions suggest in [4] an introduction of Orlicz-Sobolev spaces for those problems as (1.1), (1.2).

In this paper we consider the initial-boundary value problems for evolution equations of the form

(1.3) $$\frac{\partial u}{\partial t}+Au=f$$

with conditions

(1.4) $$u(x,0)=u_0(x)$$

(1.5) $$u|_{\partial\Omega}=0$$

in some Orlicz-Sobolev spaces where Au are of a growth not equivalent to any power and are similar to (1.2). Our equations (1.3) furnish a simple example:

$$\frac{\partial u}{\partial t}-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\left|\frac{\partial u}{\partial x_i}\right|^{p-2}\frac{\partial u}{\partial x_i}\right)\ln\left(\left|\frac{\partial u}{\partial x_i}\right|+1\right)=f,\qquad p\geqslant 2.$$

2. Preliminaries. In this section we give some necessary definitions and lemmas from Orlicz spaces which are referred to in [3], [2]. We call a function an N-function if it admits of the representation

(2.1) $$M(\xi)=\int_0^{|\xi|}p(t)dt$$

where the function $p(t)$ is upper-continuous for $t\geqslant 0$, positive for $t>0$ and non-decreasing with conditions

$$p(0)=0,\qquad \lim_{t\to\infty}p(t)=\infty.$$

$M(\xi)$, a real-valued function on R^1, is an N-function if and only if $M(\xi)$ is a continuous even function which is convex, increasing for $u\geqslant 0$ and satisfies