106. *The Whitehead Theorems in Shape Theory*

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§1. **Introduction.** Recently the notion of shape which was originally introduced by K. Borsuk [2] for compact metric spaces has been extended to the case of topological spaces by S. Mardešić [3]. In this paper we shall use the notion of shape in the sense of Mardešić [3]. As in our previous paper [6], let $\pi_n(X, A, x_0)$ be the n-th (Čech) homotopy pro-group of a pair (X, A, x_0) of pointed topological spaces and $H_n(X, A)$ the n-th (Čech) homology pro-group of a pair (X, A) of topological spaces. For a continuous map $f:(X, A, x_0)\to(Y, B, y_0)$ let us denote by $\pi_k(f)$ or $H_k(f)$ the induced morphism of the k-th homotopy or homology pro-groups.

In this paper we shall establish the following theorems as analogues of the classical Whitehead theorems.

Theorem 1. *Let (X, x_0) and (Y, y_0) be connected pointed spaces and let $f:(X, x_0)\to(Y, y_0)$ be a continuous map. For $n\geqq 2$ let us consider the following conditions.*

(i) $\pi_k(f):\pi_k(X, x_0)\to\pi_k(Y, y_0)$
is an isomorphism for $1\leqq k<n$ and an epimorphism for $k=n$.

(ii) $H_k(f): H_k(X, x_0)\to H_k(Y, y_0)$
is an isomorphism for $1\leqq k<n$ and an epimorphism for $k=n$.

Then (i) *implies* (ii), *and conversely, if $\pi_1(X, x_0)=0$ and $\pi_1(Y, y_0)=0$,* (ii) *implies* (i).

Theorem 2. *Let (X, x_0) and (Y, y_0) be connected pointed spaces of finite dimension and let $n_0=\max(1+\dim X, \dim Y)$. If $f:(X, x_0)\to(Y, y_0)$ is a continuous map such that the induced morphism $\pi_k(f)$: $\pi_k(X, x_0)\to\pi_k(Y, y_0)$ is a bimorphism for $1\leqq k<n_0$ and an epimorphism for $k=n_0$, then f induces a shape equivalence.*

In [4] Mardešić deduced the conclusion of Theorem 2 under a condition that $\pi_k(f)$ is a bimorphism for $1\leqq k\leqq n_0$ and an epimorphism for $k=n_0+1$. For the case of compact metric spaces the same result as in Mardešić [4] was obtained earlier by M. Moszyńska [9].

§2. **Preliminaries.** Let $f:(X, x_0)\to(Y, y_0)$ be a continuous map of pointed topological spaces. Let (Z, x_0) be the mapping cylinder of f which is obtained from the topological sum $(X\times I)\cup Y$ (where I is the closed unit interval $[0, 1]$ in the real line) by identifying $(x, 1)$ with $f(x)$ for $x\in X$ and by shrinking $(x_0\times I)\cup\{y_0\}$ to a point which we denote by