105. *The Hurewicz Isomorphism Theorem on Homotopy and Homology Pro-Groups*

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§1. **Introduction.** Let (X, A, x_0) be a pair of pointed topological spaces. Let $\{\mathfrak{U}_\lambda \mid \lambda \in \Lambda\}$ be the family of all locally finite normal open covers of X such that each $\mathfrak{U}_\lambda$ has exactly one member containing x_0. Then we have an inverse system $\{(X_\lambda, A_\lambda, x_{0\lambda}), [p_{\lambda\lambda'}], \Lambda\}$ in the pro-category of the homotopy category of pairs of pointed CW complexes by taking the nerves of $\mathfrak{U}_\lambda$ and $\mathfrak{U}_\lambda \cap A$, by ordering Λ by means of refinements of covers, and by taking the homotopy classes of canonical projections. We call this inverse system the Čech system of (X, A, x_0). The Čech system of (X, A) is defined similarly by using all locally finite normal open covers of X.

We define the n-th (Čech) homotopy pro-group $\pi_n(X, A, x_0)$ to be a pro-group $\{\pi_n(X_\lambda, A_\lambda, x_{0\lambda}), \pi_n(p_{\lambda\lambda'}), \Lambda)\}$ $(n \geqq 2)$; $\pi_1(X, A, x_0) = \{\pi_1(X_\lambda, A_\lambda, x_{0\lambda}), \pi_1(p_{\lambda\lambda'}), \Lambda\}$ is considered as a pro-object in the category of pointed sets and base-point preserving maps.

The n-th (Čech) homology pro-group $H_n(X, A)$ with coefficients in the additive group of integers is defined similarly by using the Čech system of (X, A). Since $\{\mathfrak{U}_\lambda \mid \lambda \in \Lambda\}$ described above is cofinal in the family of all locally finite normal open covers of X, the inverse system $\{H_n(X_\lambda, A_\lambda), H_n(p_{\lambda\lambda'}), \Lambda\}$ is isomorphic to $H_n(X, A)$ in the category of pro-groups. Hence, the set of the Hurewicz homomorphisms $\Phi_n(X_\lambda, A_\lambda, x_{0\lambda}): \pi_n(X_\lambda, A_\lambda, x_{0\lambda}) \rightarrow H_n(X_\lambda, A_\lambda)$ for $\lambda \in \Lambda$ determines a morphism $\Phi_n(X, A, x_0): \pi_n(X, A, x_0) \rightarrow H_n(X, A)$ in the category of pro-groups, which we shall call the Hurewicz morphism.

A subspace A of a space X is said to be P-embedded in X if every locally finite normal open cover of A has a refinement which can be extended to a locally finite normal open cover of X. If A is P-embedded in X, $\{(A_\lambda, x_{0\lambda}), [p_{\lambda\lambda'} \mid (A_{\lambda'}, x_{0\lambda'})], \Lambda\}$, which is obtained from the Čech system of (X, A, x_0), is isomorphic to the Čech system of (A, x_0). A pro-group $G = \{G_\lambda, \phi_{\lambda\lambda'}, \Lambda\}$ is a zero-object, $G = 0$ in notation, if G is isomorphic to a pro-group consisting of a single trivial group, or equivalently, if for each $\lambda \in \Lambda$ there is $\lambda' \in \Lambda$ with $\lambda < \lambda'$ such that $\phi_{\lambda\lambda'} = 0$.

In this paper we shall establish the following analogue of the Hurewicz isomorphism theorem.

Theorem 1. *Let (X, A, x_0) be a pair of pointed, connected, topological spaces such that $\pi_k(X, A, x_0) = 0$ for k with $1 \leqq k \leqq n$ $(n \geqq 1)$. Then*