# 100. *Note on Locally Definable Classes of Structures*

By Takao KASHIWAGI
Yamaguchi University

(Comm. by Kenjiro SHODA, M. J. A., Sept. 12, 1974)

In this note we shall study certain generalizations of the following property (♯) for a class $K$ of structures:

(♯) *A structure $\mathfrak{A}$ is in $K$ whenever each finite relative partial substructure of $\mathfrak{A}$ defined by each finite number of operations and relations can be embedded in some structure in $K$.*

This property (♯) was introduced by Mal'cev ([2] and [3]), and he called a class having the property (♯) *locally definable.* On the property (♯) the following is known (Tarski [6] and Mal'cev [4; p. 138]):

(∗) *A class $K$ is universal if and only if $K$ has the property* (♯).

Tarski [7] gave two analogous theorems. One is for an infinitary language without operation symbols and the other is for an infinitary language without relation symbols. We shall make the similar investigation for an infinitary language $L_{\alpha\beta}$ with an arbitrary number of finitary operation and relation symbols. We introduce a notion of $[\alpha, \beta]$-local definability as an analogue of local definability. Our main theorem below, which is a generalization of the theorems of Tarski ([7; Theorems 1 and 2]), shows that every $[\alpha, \beta]$-locally definable class can be characterized as a class defined by a set of universal sentences in prenex form of the infinitary language $L_{\alpha\beta}$. A generalization of (∗) is stated in Corollary of this theorem.

There are two similar works for classes of algebras in [5] and for classes of relational systems in [1]. The former ([5; Theorem 2 ((i)$\Longleftrightarrow$(ii))]) follows immediately from our theorem. But it seems that the latter does not follow from our results, because [1] deals with classes defined by universal sentences of $L_{\alpha\beta}$ which are not necessarily in prenex form.

The letters $\lambda, \mu, \nu, \xi, \zeta$ will be used to denote ordinals, and $\alpha, \beta$ will be used to denote cardinals (initial ordinals). The cardinality of a set $A$ is denoted by $|A|$. $\mathfrak{A}=\langle A; \{f_A : f \in F\}, \{r_A : r \in R\}\rangle$ is called a structure, if $A$ is a nonvoid set, and there are maps $n : F \to \omega$ and $m : R \to \omega$ such that for $f \in F, f_A$ is an $n(f)$-ary operation on $A$ and for $r \in R, r_A$ is an $m(r)$-ary relation on $A$. $A$ is called the universe of $\mathfrak{A}$. The sequence $\tau=\langle F, n, R, m\rangle$, uniquely determined by $\mathfrak{A}$, is called the similarity type of $\mathfrak{A}$. The type $\tau$ is fixed throughout this note. Capital German letters denote structures and the corresponding capital Roman