96. *Fourier Transform of Banach Algebra Valued Functions on Group. II*[*)]

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The next theorem is a generalization of the theorem in the previous paper.

Theorem. *Let h be a continuous mapping of $L^1(G\to A)$ into B with the following properties;*

(1) $h(af+bg)=ah(f)+bh(g)$ *for any complex numbers a, b, and f, $g\in L^1(G\to A)$,*

(2) $h(f*g)=h(f)\cdot h(g)$ *for $f,g\in L^1(G\to A)$,*

(3) *for any $\varepsilon>0$ there exists $f_\varepsilon\in L^1(G\to A)$ such that $\|h(f_\varepsilon)-1\|_B<\varepsilon$.*

Then there exist a homomorphism α of A into B and a bounded continuous homomorphism φ of G into $C_B(\alpha(A))$ such that

$$h(f)=\int_G \varphi(x)\alpha(f(x))dx, \qquad \text{for } f\in L^1(G\to A),$$

where $C_B(\alpha(A))$ means the set of all elements of B that commute with every element in the range of α.

Proof. By the property (3), there exists $f_1\in L^1(G\to A)$ such that $h(f_1)^{-1}$ exists in B. For this f_1 and for any fixed $f\in L^1(G\to A)$, by Proposition 4, there exists a sequence $\{E_n\}$ of measurable sets in G such that

$$|||m(E_n)^{-1}\chi_{E_n}*f_1-f_1|||<1/n,$$
$$|||m(E_n)^{-1}\chi_{E_n}*f-f|||<1/n, \qquad (n=1,2,\cdots).$$

Then, for $a\in A$,

$$\|m(E_n)^{-1}h(\chi_{E_n}*af_1)-h(af_1)\|_B=\|m(E_n)^{-1}h(a\chi_{E_n})h(f_1)-h(af_1)\|_B \leqq \|h\|\cdot\|a\|/n,$$

which vanishes as n tends to ∞.

We put $\alpha(a)=\lim_{n\to\infty} m(E_n)^{-1}h(a\chi_{E_n})=h(af_1)h(f_1)^{-1}$. Replacing f_1 by f in the inequality above, we get $h(af)=\alpha(a)h(f)$.

Since the definition of α does not depend on the choice of $\{E_n\}$, $h(af)=\alpha(a)h(f)$ holds good for every $f\in L^1(G\to A)$.

We show α is a homomorphism.

$$\alpha(ab)=\alpha(ab)h(f_1)h(f_1)^{-1}=h(abf_1)h(f_1)^{-1}=\alpha(a)\alpha(b)h(f_1)h(f_1)^{-1} =\alpha(a)\alpha(b).$$

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