95. *Structure of Cohomology Groups Whose Coefficients are Microfunction Solution Sheaves of Systems of Pseudo-Differential Equations with Multiple Characteristics. I*

By Masaki KASHIWARA,*) Takahiro KAWAI,**) and Toshio OSHIMA***)

(Comm. by Kôsaku YOSIDA, M. J. A., Sept. 12, 1974)

In this note we investigate the structure of cohomology groups whose coefficients are the microfunction solution sheaf of a system $\mathcal{M}$ of linear pseudo-differential equations, when the characteristic variety V of $\mathcal{M}$ has singularities of rather limited type, i.e., V has the form $V_1 \cup V_2$, where at least either one of them is regular. The precise conditions on V and $\mathcal{M}$ will be given in the below. Our method consists in two steps: firstly we investigate the structure of the system $\mathcal{M}$ itself in the complex domain and secondly we calculate the cohomology groups by making use of a special expression of sheaf $\mathcal{C}$ of microfunctions. In the first step we rely on the results obtained in Chapter II of Sato-Kawai-Kashiwara [9] (hereafter referred to as S-K-K [9]) and in the second step we resort to the theory of boundary value problems for elliptic system of linear differential equations, which is expounded in Kashiwara-Kawai [7]. These arguments will be also used in Kashiwara [6].

Note that the investigation of cohomology groups whose coefficients are the microfunction solution sheaf of the system of pseudo-differential equations are fully investigated in S-K-K [9] under the assumption that its characteristic variety is regular, that is, V is non-singular as a variety and $\omega|_V \neq 0$ for the canonical 1-form ω of the contact manifold in which V lies. See also Oshima [8] for the case of maximally degenerate system. We also note that great efforts have recently been made to clarify the situation we encounter in the last half part of this note by Grušin [5], Treves [12], Boutet de Monvel-Treves [1], [2], Sjöstrand [10], Folland [3], Folland-Stein [4], Taira [11], Boutet de Monvel [13], and others in the case of determined systems, though they only discuss the (micro-local) C^∞-regularity with the exception of Grušin [5], where the (global) C^∞-regularity is also discussed.

*) Mathematical Institute, Nagoya University.
**) Research Institute for Mathematical Sciences, Kyoto University.
***) Department of Mathematics, University of Tokyo.