94. On Strongly Pseudo-Convex Manifolds

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By a strongly pseudo-convex (s.p.c) manifold we mean the abstract model (cf. Kohn [2]) of a s.p.c. real hypersurface of a complex manifold. The main aim of this note is to announce some theorems on compact s.p.c. manifolds M, especially on the cohomology groups $H^{p,q}(M)$ due to Kohn-Rossi [3] and the holomorphic de Rham cohomology groups $H_0^k(M)$ (see Theorems 1, 2). We also apply Theorem 2 to the study of isolated singular points of complex hypersurfaces (see Theorem 4).

Throughout this note we always assume the differentiability of class C^∞. Given a fibre bundle E over a manifold M, $\Gamma(E)$ denotes the set of differentiable cross sections of E.

1. S.p.c. manifolds. Let M' be an n-dimensional complex manifold and M a real hypersurface of M'. Let T' (resp. T) be the complexified tangent bundle of M' (resp. of M). Denote by S' the subbundle of T' consisting of all tangent vectors of type $(1,0)$ to M' and, for each $x \in M$, put $S_x = T_x \cap S'_x$. Then we have $\dim_C S_x = n-1$ and hence the union $S = \bigcup_x S_x$ forms a subbundle of T. It is easy to see that S satisfies

1) $S \cap \bar{S} = 0$,

2) $[\Gamma(S), \Gamma(S)] \subset \Gamma(S)$.

By 1), the sum $P = S + \bar{S}$ is a subbundle of T. Consider the factor bundle $Q = T/P$ and denote by ϖ the projection of T onto Q. For each $x \in M$, define an Q_x-valued quadratic form H_x on S_x, the Levi form at x, by $H_x(X_x) = \varpi([X, \bar{X}]_x)$ for all $X \in \Gamma(S)$. Then M is, by definition, s.p.c. if S satisfies

3) the Levi form H_x is definite at each $x \in M$.

Let M be a (real) manifold of dimension $2n-1$. Suppose that there is given an $(n-1)$-dimensional subbundle S of the complexified tangent bundle T of M. Then S is called a s.p.c. structure if it satisfies conditions 1), 2) and 3) stated above, and the manifold M together with the structure is called a s.p.c. manifold.

2. The cohomology groups $H^{p,q}(M)$, $H_0^k(M)$ and $H_*^{p,q}(M)$. Let M be a s.p.c. manifold of dimension $2n-1$ and S its s.p.c. structure. Let $\{\mathcal{A}^k, d\}$ be the de Rham complex of M with complex coefficients.