91. *Extension Theorems for Kähler Metrics*

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Let X be a complex manifold and let $\hat{X}$ denote a monoidal transform of X of which the center is a point. The folowing proposition is well-known:

If X admits a Kähler metric, then $\hat{X}$ also admits a Kähler metric.

In this note we shall prove an extension theorem for Kähler metrics of which the converse of the above proposition is a corollary. Moreover we shall show a similar extension theorem for a certain type of branched coverings.

1. Formulation of the results. In this section we denote by X a complex manifold and let $D_r^n=\{z\in \boldsymbol{C}^n \mid z<r\}$.

Proposition A. *Assume that D_r^n-0 has a Kähler form ω. Then D_r^n admits a Kähler form $\tilde{\omega}$ such that $\tilde{\omega}=\omega$ on $D_r^n-D_{2r/3}^n$.*

Corollary. *Let P denote a point on X. If $X-P$ is a Kähler manifold, then X is also a Kähler manifold.*

Let $\tilde{D}=D_r^1\to D=D_{r^m}^1$ be the m-fold branched covering defined by the mapping $z\to z^m$, and let Γ denote the covering transformation group of $\tilde{D}$ with respect to D. Moreover let $p: X\to D$ be a surjective proper smooth holomorphic mapping, $\tilde{X}=X\times_D\tilde{D}$, and denote by π the induced covering map: $\tilde{X}\to X$. The group Γ acts on $\tilde{X}$ in an obvious manner.

Proposition B. *If $\tilde{X}$ has a Γ-invariant Kähler metric $\tilde{\omega}$, then X admits a Kähler metric ω such that $\tilde{\omega}=\pi^*\omega$ on $\pi^{-1}p^{-1}(D-D_{(2r/3)^m}^1)$.*

Corollary. *Let Δ and $\tilde{\Delta}$ be compact Riemann surfaces, X a compact complex manifold of dimension n, and let $p: X\to\Delta$ be a fibre manifold. Moreover let $\pi:\tilde{\Delta}\to\Delta$ be a finite Galois covering. Let $\tilde{X}$ denote the normalization of the fibre product $X\times_\Delta\tilde{\Delta}$. Assume that the induced covering $\tilde{X}\to X$ has its branch locus on regular fibres of $p: X\to\Delta$. Then X is a Kähler manifold if and only if $\tilde{X}$ is a Kähler manifold.*

2. Proof of Proposition A. By $\mathcal{D}$ and $\mathcal{F}$ we denote, respectively, the sheaves of differentiable functions and differentiable d-closed $(1,1)$-forms. We have a natural exact sequence of sheaves:

$$0\longrightarrow\mathcal{O}+\bar{\mathcal{O}}\longrightarrow\mathcal{D}\xrightarrow{\sqrt{-1}\partial\bar{\partial}}\mathcal{F}\longrightarrow 0.$$

Lemma 1. *Let ω be a d-closed $(1,1)$-form on $W=D_r^n-0$, $n>2$.*