140. *Double Centralizers of Torsionless Modules**)

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In this note, we make the assumption that a ring has an identity element and modules are unital. For a left R-module ${}_RM$ where R is a ring, $D=\mathrm{End}_R\,({}_RM)$ is an R-endomorphism ring of ${}_RM$ operating on the side opposite to the scalars. Then ${}_RM$ is considered as an (R, D)-bimodule. A D-endomorphism ring $Q=\mathrm{End}_D\,(M_D)$ of M_D is called a double centralizer of ${}_RM$.

Definition. Let ${}_RM$ and ${}_RU$ be left R-modules, ${}_RM$ is said to be ${}_RU$-torsionless in case for each non-zero element m of ${}_RM$, there exists an R-homomorphism ϕ of ${}_RM$ into ${}_RU$ such that $(m)\phi\neq 0$.

We say that a left R-module ${}_RM$ is torsionless if ${}_RM$ is ${}_RR$-torsionless and ${}_RN$ is faithful if ${}_RR$ is ${}_RN$-torsionless. Let Q be a double centralizer of a faithful left R-module ${}_RM$, then there exists a canonical ring monomorphism of R into Q, written as $R\hookrightarrow Q$. A faithful left R-module ${}_RM$ is said to have the double centralizer property if $R=Q$, where Q is a double centralizer of ${}_RM$.

Definition. A ring R is left QF-1 if every faithful left R-module has the double centralizer property.

QF-1 rings were first described by R. M. Thrall (1948 [4]) and have been examined by many authors. It was proved that the double centralizer of a faithful torsionless left R-module is a rational extension of R_R. Furthermore the double centralizer of a dominant left R-module is a maximal right quotient ring of R (see T. Kato [1] and H. Tachikawa [3]). In the section 1, the next theorem is proved.

Theorem. *Let R be a ring with minimum condition and U be the intersection of all left faithful two-sided ideals of R. Then U is also a left faithful two-sided ideal of R and the double centralizer of ${}_RU$ is a maximal right quotient ring of R.*

In the section 2, we shall prove that for a given faithful left R-module ${}_RM$, ${}_RM$ has the double centralizer property if and only if ${}_KKe$ has the double centralizer property, where

$$K=\begin{pmatrix} R & M \\ \mathrm{Hom}_R\,({}_RM, {}_RR) & \mathrm{End}_R\,({}_RM)\end{pmatrix} \quad \text{and} \quad e=\begin{pmatrix} 0 & 0 \\ 0 & 1\end{pmatrix}\in K.$$

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