135. *Direct Sum of Strongly Regular Rings and Zero Rings*

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1. Introduction. In [5] F. Szász investigated a class of rings, called P_1-rings, which coincides with the class of strongly regular rings in the absence of nilpotent elements. He showed that any P_1-ring is a subdirect sum of some zero rings of additive rank one and some division rings. In this paper, we shall give several characterizations of P_1-rings, in particular, it will be shown that any P_1-ring is a direct sum of a strongly regular ring and a zero ring. We also explore other generalizations of strongly regular rings and apply them to some commutatively theorems.

2. P_1-rings. Definition 1. A ring R is called a P_1-ring if $aR = aRa$ for each a in R.

We summarize here some of the results in [5] about P_1-rings.

Theorem 0. *Let R be a P_1-ring. Then*

(i) *$aR = aRa^n$ for any positive integer n and $NR = 0$ where N denotes the set of nilpotent elements of R.*

(ii) *R is strongly regular if and only if R has no nonzero nilpotent elements.*

Now we give a characterization of P_1-rings, but first a lemma is needed.

Lemma 1. *Let R be a P_1-ring. Then $ab = 0$ implies $ba = 0$ for any a, b in R.*

Proof. Suppose $ab = 0$. Then $baba = 0$ implies that ba is in N and from (i) of Theorem 0, $baR = 0$. R is P_1 implies that $ba = brb$ for some r in R. Hence $bar = brbr = 0$. Thus br is in N and $brR = 0$. Consequently $ba = brb = 0$.

Theorem 1. *A ring R is a P_1-ring if and only if*

(i) *$N \subseteq C$, where C denotes the center of R,*

(ii) *$E \subseteq C$, where E denotes the set of idempotents,*

(iii) *$NR = 0$,*

(iv) *R/N is strongly regular.*

Proof. Suppose R is a P_1-ring. If x is in N, then $xR = 0$. By Lemma 1, $Rx = 0$ and hence $N \subseteq C$. Now let $e = e^2$ be in R. Then for any x in R, $e(ex - x) = 0$ implies that $(ex - x)e = 0$ and $exe = xe$. Sim-

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