128. *Energy Inequalities and Finite Propagation Speed of the Cauchy Problem for Hyperbolic Equations with Constantly Multiple Characteristics*

By Kiyoshi YOSHIDA
Faculty of Science, Hiroshima University

(Comm. by Kôsaku YOSIDA, M. J. A., Oct. 12, 1974)

Let $P(t,x;D_t,D_x)$ be a differential polynomial defined in a domain $\Omega=[0,T]\times R^n$, $T>0$, of the form $P(t,x;D_t,D_x)=D_t^m+\sum_{j=0}^{m-1}\sum_{j+|\nu|\leqq m} a_{j,\nu}(t,x)D_t^jD_x^\nu$, $a_{j,\nu}\in\mathcal{B}(\Omega)$, with $D_t=-i\partial/\partial t$ and $D_x=(-i\partial/\partial x_1,\cdots,-i\partial/\partial x_n)$, and let us consider the Cauchy problem:

$$(1)\qquad \begin{cases} Pu=f & \text{in } (0,T)\times R^n \\ D_t^ju(0,x)=u_j, & j=0,1,\cdots,m-1 \end{cases}$$

for given $f\in C^\infty(\Omega)$ and $u_j\in C^\infty(R^n)$. It is well-known that the characteristic roots are real if the Cauchy problem is well-posed in C^∞ (cf. [3]). In the present note we study a sufficient conditions for the problem (1) to be well posed when charateristics are real and have constant multiplicity. Concerning this problem, S. Mizohata and Y. Ohya [4], [5] obtained a necessary and sufficient condition when the multiplicity is less than 2, and Y. Ohya [6] studied a sufficient condition when the multiplicity is less than 3. Recently, J. Chazarain [1] discusses the case of the arbitrary multiplicity by making use of the theory of Fourier integral operators. Our arguments seem to be different from his.

1. E. E. Levi's condition and the main theorem. Let the principal part P_m of P be written as $P_m(t,x;\tau,\xi)=\prod_{j=0}^{r}(\tau-\lambda_j(t,x;\xi))^{l_j}$, and assume that $\lambda_j(t,x;\xi)$, $1\leqq j\leqq r$, are real for $\xi\in R^n-\{0\}$ and $\inf_{(t,x)\in\Omega,|\xi|=1}|\lambda_j(t,x;\xi)-\lambda_k(t,x;\xi)|\geqq d>0$, $j\neq k$. Moreover, without loss of generality we may assume that $l_1=l_2\cdots=l_{r_1}>l_{r_1+1}=\cdots=l_{r_2}>\cdots>\cdots=l_r$, and put $l=l_1$. Let $\theta(\xi)$ be a C^∞-function such that $\theta(\xi)=0$ for $|\xi|\leqq\frac{1}{4}$, and $\theta(\xi)=1$ for $|\xi|\geqq\frac{1}{2}$, and define $\lambda_j(t,x;D_x)$ by

$$\lambda_j(t,x;D_x)\phi=\frac{1}{(2\pi)^n}\int\lambda_j(t,x;\xi)\theta(\xi)\hat{\phi}(\xi)e^{ix\cdot\xi}d\xi.$$

Then $\lambda_j(t,x;D_x)$ are pseudo-differential operators of class $\mathcal{E}_t(S^1)$. Here we have denoted by S^p the set of C^∞-functions $h(x,\xi)$ on $R^n\times R^n$ such that

$$|D_x^\mu D_\xi^\nu h(x,\xi)|\leqq C_{\mu,\nu}(1+|\xi|^2)^{1/2(p-|\nu|)}\qquad \text{in } R^n\times R^n,$$