164. *Defect Relations and Ramification*

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In this paper we generalize the theory of ramified values in the Nevanlinna theory ([4], [7]) to the case of equidimensional holomorphic maps from C^n into projective algebraic manifolds and we prove variants of a defect relation of Carlson and Griffiths [1]. (See also [3], [9].)

1. Let W be a projective algebraic manifold of dimension n and L a line bundle on W. Iitaka [5] defined the *L-dimension* $\kappa(L, W)$ of W, which is roughly the polynomial order of $\dim H^0(W, \mathcal{O}(mL))$ as a function of m, as follows. If there is a positive integer m_0 such that $\dim H^0(W, \mathcal{O}(m_0L))>0$, we have the following estimate:

$$\alpha m^\kappa \leqq \dim H^0(W, \mathcal{O}(mm_0L)) \leqq \beta m^\kappa,$$

for large integer m and positive constants α, β, where κ is a non-negative integer uniquely determined by L. Then we define $\kappa(L, W)=\kappa$. In the other case, we put $\kappa(L, W)=-\infty$. In particular, $\kappa(L, W)=n$ if and only if

$$\limsup_{m\to+\infty} m^{-n} \dim H^0(W, \mathcal{O}(mL))>0.$$

For a divisor D on W, denote by $[D]$ the line bundle associated with D. Define $\kappa(D, W)=\kappa([D], W)$. By $L_1+\cdots+L_k$, we mean the tensor product $L_1\otimes\cdots\otimes L_k$ of line bundles $L_1, \cdots, L_k$. Moreover we shall consider linear combinations of line bundles: $L=q_1L_1+\cdots+q_kL_k$, with rational numbers $q_1, \cdots, q_k$. Define $\kappa(L, W)$ to be $\kappa(mL, W)$ for any positive integer m such that each mq_i is an integer.

2. We shall consider holomorphic maps $f: C^n \to W$, and assume that f is *non-degenerate*, i.e., the Jacobian J_f of f does not vanish identically. Let D be an effective divisor on W. Denote by $\operatorname{Supp}(f^*D)$ the support of the divisor f^*D. Namely, if $f^*D=\sum_s m_s Z_s$, with Z_s irreducible, we put $\operatorname{Supp}(f^*D)=\sum_s Z_s$. Let $(z_1, \cdots, z_n)$ be holomorphic coordinates in C^n, and let $B[r]$ denote a ball of radius r: $B[r]=\{z\in C^n \mid \|z\|<r\}$, where $\|z\|^2=|z_1|^2+\cdots+|z_n|^2$. For a set X in C^n, let $X[r]=X\cap B[r]$. We use the following notations:

$$\psi=(2\pi)^{-1}\sqrt{-1}\partial\bar\partial \log\|z\|^2,$$

$$N(D, r)=\int_0^r \left(\int_{f^*D[t]} \psi^{n-1}\right) t^{-1}dt,$$

$$\bar N(D, r)=\int_0^r \left(\int_{\operatorname{Supp}(f^*D)[t]} \psi^{n-1}\right) t^{-1}dt,$$