190. *Characters of Finite Groups with Split (B, N)-Pairs*

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§ 1. In our previous paper [4], we discussed the irreducibility of characters of the finite general unitary group $GU(n, q^2)$ induced by those of a direct product of the finite general linear group $GL(k, q^2)$ and $GU(n-2k, q^2)$. Recently we were suggested by Professor C. W. Curtis that one would be able to get a similar result for finite groups with split (B, N)-pairs. Using the results of intersections of parabolic subgroups in a paper by Curtis [2], we could generalize the result in our paper [4]. Note that this is a special case of Theorem 3.5 due to Curtis [2].

I wish to thank Professor Curtis for his suggestion to me on this problem and also for the generous use of his preprint [2].

By a character of a group, we mean a rational integral combination of its complex irreducible characters. Standard notations for finite group theory and character theory will be used.

Let G be a finite group with a split (B, N)-pair of characteristic p, for some prime p, and Coxeter system (W, R). Let P_J be a standard maximal parabolic subgroup of G, L_J the standard Levi factor of P_J for some $J \subseteq R$. Then P_J has a semi-direct decomposition $P_J = L_J V_J$ of $V_J = O_p(P_J)$ by L_J, which we call the Levi decomposition of P_J. If χ is an irreducible character of L_J, then we can extend χ to an irreducible character $\tilde{\chi}$ of P_J, by putting $\tilde{\chi}(lv) = \chi(l)$ for $l \in L_J, v \in V_J$. We shall now prove the following

Theorem. *Let $W_{J,J}$ be the set of distinguished (W_J, W_J)-double coset representatives of W. Assume that* (i) *χ is not a self-conjugate and* (ii) *no kernel of irreducible constituents of the restriction of χ to $L_J \cap {}^wP_J$ contains $L_J \cap {}^wV_J$ whenever $L_J \neq {}^wL_J$ for $w \in W_{J,J}$. Then the character $\tilde{\chi}^G$ of G induced by $\tilde{\chi}$ is irreducible.*

In order to prove this theorem, we must calculate the scalar product $(\tilde{\chi}^G, \tilde{\chi}^G)_G$. To do this, it will be necessary to derive some informations of parabolic subgroups. In § 2, we shall state several results about intersections of parabolic subgroups due to Curtis [2]. The theorem is proved in § 3. The proof is a simple combination of lemmas in § 2, § 3.

§ 2. Let (G, B, N, W, R) be as in § 1. Then W is isomorphic to the Weyl group $W(\Delta)$ of a uniquely determined root system Δ, such