187. *Denseness of Singular Densities*

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Consider a 2-form $P(z)dxdy$ on an open Riemann surface R such that the coefficients $P(z)$ are nonnegative locally Hölder continuous functions of local parameters $z=x+iy$ on R. Such a 2-form $P(z)dxdy$ will be referred to as a *density* on R. We shall call a density P *singular* if any nonnegative C^2 solution u of the elliptic equation

(1) $\Delta u(z)=P(z)u(z)$ (i.e. $d^*du(z)=u(z)P(z)dxdy$)

on R has the zero infimum, i.e. $\inf_{z\in R} u(z)=0$. We denote by $D=D(R)$ and $D_S=D_S(R)$ the set of densities and singular densities on R, respectively. According to Myrberg [2], (1) always possesses at least one strictly positive solution for any open Riemann surface R. In connection with the existence of Evans solution, Nakai [5] showed that $D_S\neq\emptyset$ for any open Riemann surface R. The purpose of this note is to show that D_S is not only nonvoid but also contains sufficiently many members in the following sense: D_S is dense in D with respect to the metric

$$\rho(P_1,P_2)=\left(\int_R |P_1(z)-P_2(z)|\,dxdy\right)^*$$

on D, where $a^*=a/(1+a)$ for nonnegative numbers and $\infty^*=1$. Namely, we shall prove the following

Theorem. *The subspace $D_S(R)$ of singular densities is dense in the metric space $(D(R),\rho)$ for any open Riemann surface R.*

Proof. We only have to show that for any $P\in D$ and any $\eta>0$, there exists a $Q\in D_S$ such that

(2) $$\int_R |P(z)-Q(z)|\,dxdy\leq\eta.$$

Our proof goes on an analogous way to [5]. Let $(\{z_j\}, \{U_j\}, \{\eta_j\})$ $(j=1,2,\cdots)$ be a system such that $\{z_j\}$ is a sequence of points in R not accumulating in R, U_j are parametric disks on R with centers z_j such that $\overline{U}_j\cap\overline{U}_k=\emptyset$ $(j\neq k)$, and $\{\eta_j\}$ is a sequence with $\eta_j>0$ and $\sum_{j=1}^{\infty}\eta_j=\eta$. Furthermore we denote by V_j the concentric parametric disk $|z|<\rho_j=\exp(-4\pi/\eta_j)$ of U_j $(j=1,2,\cdots)$. Let $G(z,\zeta)$ be the Green's function on $S=R-\bigcup_{j=1}^{\infty}\overline{V}_j$ for (1). Fix a point $z_0\in S$ and set

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