

185. An Oscillation Theorem for Differential Equations with Deviating Argument

By Takaši KUSANO^{*)} and Hiroshi ONOSE^{**)}

(Comm. by Kôzaku YOSIDA, M. J. A., Dec. 12, 1974)

In this paper we are concerned with the oscillatory behavior of bounded solutions of the following differential equations with deviating argument

$$(A) \quad [r(t)x^{(n-1)}(t)]' + a(t)f(x(g(t))) = b(t),$$

$$(B) \quad [r(t)x'(t)]^{(n-1)} + a(t)f(x(g(t))) = b(t).$$

For these equations the following conditions are assumed to hold:

(a) $a, b \in C[R_+, R]$, $R_+ = (0, \infty)$, $R = (-\infty, \infty)$;

(b) $r \in C[R_+, R_+]$, and $\int_{t_0}^{\infty} r^{-1}(t)dt = \infty$;

(c) $f \in C[R, R]$, $yf(y) > 0$ for $y \neq 0$ and $f(y)$ is nondecreasing;

(d) $g \in C[R_+, R_+]$, and $\lim_{t \rightarrow \infty} g(t) = \infty$.

We shall consider only those solutions of (A) and (B) which exist on some half-line $[t_0, \infty)$, where t_0 may depend on the particular solution, and are nontrivial in any neighborhood of infinity. Such a solution is called oscillatory if it has arbitrarily large zeros; otherwise it is called nonoscillatory.

In case $a(t)$ is eventually nonnegative, oscillation and nonoscillation criteria for (A), (B) or related equations have been extensively developed during the past few years. (See e.g. Kusano and Onose [2], [3], Ševelo and Vareh [4], Singh [5].) However, much less is known about the case when $a(t)$ is allowed to oscillate, that is, to take both positive and negative values for arbitrarily large t . The only reference devoted to the latter case is (to our knowledge) the paper by Travis [6] in which it is shown by an example that the well-known Wintner-Leighton oscillation theorem for second order ordinary differential equations cannot directly be extended to the corresponding differential equations with functional argument.

In this paper an attempt will be made to establish an oscillation criterion for bounded solutions of the equations (A) and (B) with oscillating coefficient $a(t)$. Our result is an extension of a theorem of Kartsatos [1, Theorem 1] on the bounded oscillation of second order ordinary differential equations.

^{*)} Department of Mathematics, Hiroshima University.

^{**)} Department of Mathematics, Ibaraki University.