183. *On Certain Complex Structures on the Product of Two Odd Dimensional Spheres*

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§ 1. The purpose of this note is to report our recent results on the complex structures on the product of two spheres of odd dimensions. The details will appear elsewhere.

Calabi and Eckmann [1] constructed a complex structure on $S^{2p-1} \times S^{2q-1}$ $(p,q \geqq 2)$. We will show that, under certain additional conditions, a complex manifold homeomorphic to $S^{2p-1} \times S^{2q-1}$ is a finite abelian branched covering of a submanifold in the above-mentioned Calabi-Eckmann manifold, where p and q are greater than one.

Throughout this note, $\mathcal{O}$ (or $\mathcal{O}_X$) denotes the structure sheaf of a complex manifold X, $q = q(X) = \dim_C H^1(X, \mathcal{O})$ its irregularity, and $b_\nu = b_\nu(X)$, the ν-th Betti number.

§ 2. **The construction of Calabi-Eckmann manifolds.** We construct a Calabi-Eckmann manifold as follows: For each $t \in C$, let g_t^λ be the biholomorphic automorphism of $(C^p - (o)) \times (C^q - (o))$ which maps (z, w) to $(z \exp(t), w \exp(\lambda t))$, where λ is a fixed complex number with $\operatorname{Im} \lambda \neq 0$. Let G be the one-parameter complex Lie group consisting of g_t^λ's. G operates freely and properly on $(C^p - (o)) \times (C^q - (o))$. Hence by Holmann [2], we can construct the quotient manifold $M = (C^p - (o)) \times (C^q - (o))/G$, which is a compact complex manifold of dim. $(p+q-1)$. M is called a Calabi-Eckmann manifold. Note that M is a complex analytic fibre bundle over $P^{p-1} \times P^{q-1}$ of which the fibre is an elliptic curve. Moreover M has a holomorphic torus action.

§ 3. **A lemma on the structure of elliptic *n*-folds.** We need the following lemma.

Lemma. *Let V^n and $\tilde{D}^{n-1}$ be an n-dimensional complex manifold and an $(n-1)$-dimensional polydisk, respectively. Let $\pi: V^n \to \tilde{D}^{n-1}$ be a flat proper holomorphic mapping, whose general fibres are biholomorphically equivalent to a fixed elliptic curve E. Suppose that there exists a subvariety S of $\tilde{D}$ with codimension greater than* 1, *such that, outside of S, every fibre of π is a non-singular elliptic curve. Then there exists a polydisk $D \subset \tilde{D}$ such that every fibre of $\pi|_D$ is a non-singular elliptic curve having the general fibre E as a finite unramified covering, and that the singular locus, i.e., the image of the critical*

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