# 180. A Remark on q-conformally Flat Product Riemannian Manifolds

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Recently, the study of curvature structures of higher order has been developed by J. A. Thorpe, R. S. Kulkarni and many other people. Especially, Kulkarni has introduced the interesting double form *con* $\omega$ associated with the given double form $\omega$, which is a generalization of Weyl's conformal curvature tensor for the case of higher order. Also, the present first author has studied in [3] on $q$-conformal flatness for Riemannian manifolds.

The object of this paper is to investigate on the double forms in product Riemannian manifolds, and apply it to obtain a theorem on $q$-conformally flat product Riemannian manifolds. An exposition with detailed proof of Theorem 2 will be published elsewhere.

We shall assume, throughout this paper, that all manifolds are connected and all objects are of differentiability class $C^\infty$. For the terminology and notation, we generally follow [1] and [2].

**1.** In this section we shall give a brief summary of basic formulae for later use (for the details, see [2] or [3]).

Let $\Lambda^p(V)$ and $\Lambda^p(V^*)$ denote the exterior powers of a real $n$-dimensional vector space $V$ and its dual space $V^*$, respectively ($0 \leqq p \leqq n$). We consider the spaces

$$\mathcal{D}^{p,q}(V)=\Lambda^p(V^*)\otimes\Lambda^q(V^*),\quad 0\leqq p,q\leqq n,\quad \mathcal{D}(V)=\sum_{p,q=0}^{n}\mathcal{D}^{p,q}(V).$$

An element $\omega\in\mathcal{D}^{p,q}(V)$ is called *double form of type* $(p,q)$ *on* $V$, and its value on $u=x_1\wedge x_2\wedge\cdots\wedge x_p\in\Lambda^p(V)$ and $v=y_1\wedge y_2\wedge\cdots\wedge y_q\in\Lambda^q(V)$ is denoted by

$$\omega(u\otimes v)=\omega(x_1x_2\cdots x_p\otimes y_1y_2\cdots y_q).$$

$\mathcal{D}(V)$ forms an associative ring with respect to the natural "exterior multiplication $\wedge$", and we have

$$(1)\qquad \omega\wedge\theta=(-1)^{pr+qs}\theta\wedge\omega$$

for any double forms $\omega,\theta$ of types $(p,q)$, $(r,s)$, respectively. A symmetric double form of type $(p,p)$ is called the *curvature structure of order* $p$ *on* $V$, and the set of such elements is denoted by $C^p(V)$. $C(V)=\sum_{p=0}^{n}C^p(V)$ forms a commutative subring of $\mathcal{D}(V)$ called the *ring of*

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