2. *Remarks on a Totally Real Submanifold*

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§ 1. **Introduction.** K. Yano and S. Ishihara [8] and J. Erbacher [3] have determined the submanifold M of non-negative sectional curvature in the Euclidean space or in the sphere with constant mean curvature, such that M has a constant scalar curvature and a flat normal connection.

Recently, C. S. Houh [4], S. T. Yau [9], and B. Y. Chen and K. Ogiue [2] have investigated totally real submanifolds in a Kähler manifold with constant holomorphic sectional curvature c.

On the other hand, the authors [5]-[7] studied C-totally real submanifolds in a Sasakian manifold with constant ϕ-holomorphic sectional curvature. In particular, we have dealt with C-totally real submanifolds with flat normal connection in [6].

The purpose of this paper is to obtain the following:

Theorem. *Let M^n be a totally real submanifold in a Kähler manifold $\bar{M}^{2n}$. A necessary and sufficient condition in order that the normal connection is flat is that the submanifold M^n is flat.*

§ 2. **Preliminaries.** Let M^n be a submanifold immersed in a Riemannian manifold $\bar{M}^{n+p}$. Let $\langle\ ,\ \rangle$ be the metric tensor field on $\bar{M}^{n+p}$ as well as the metric tensor induced on M^n. We denote by $\bar{\nabla}$ the covariant differentiation in $\bar{M}^{n+p}$ and ∇ the covariant differentiation in M^n determined by the induced metric on M^n. Let $\mathfrak{X}(\bar{M})$ (resp. $\mathfrak{X}(M)$) be the Lie algebra of vector fields on $\bar{M}$ (resp. M) and $\mathfrak{X}^{\perp}(M)$ the set of all vector fields normal to M^n.

The Gauss-Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y), \tag{2.1}$$

$$\bar{\nabla}_X N = -A^N(X) + D_X N, \quad X, Y \in \mathfrak{X}(M), \quad N \in \mathfrak{X}^{\perp}(M), \tag{2.2}$$

where $\langle B(X, Y), N\rangle = \langle A^N(X), Y\rangle$ and $D_X N$ is the covariant derivative of the normal connection. A and B are called the second fundamental form of M.

The curvature tensors associated with $\bar{\nabla}, \nabla, D$ are defined by the followings respectively:

$$\begin{aligned} \bar{R}(X, Y) &= [\bar{\nabla}_X, \bar{\nabla}_Y] - \bar{\nabla}_{[X,Y]}, \\ R(X, Y) &= [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}, \\ R^{\perp}(X, Y) &= [D_X, D_Y] - D_{[X,Y]}. \end{aligned} \tag{2.3}$$

If the curvature tensor $R^{\perp}$ of the normal connection D vanishes, then