1. *Finiteness of Objects in Categories*

By Tadashi OHKUMA

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Introduction. The well-known notion of ultraproducts in model theory was re-defined in terms of categories in [2], and it was observed that having an injective diagonal map $d: B \to B^{\Lambda}/D$ for any set Λ and its ultrafilter D is essential for the object B to have an algebraic (finitary) structure. However, there, we dealt with only concrete categories. When we deal with sets such as objects in concrete categories, we tacitly assume that the notion of finiteness is well understood, but in order to generalize the theorems into abstract categories, we need to define the notion of finiteness in terms of categories.

We made some attempt in [3] to describe the finiteness of structure in objects in terms of categories, and thereby some of theorems in [2] were generalized. Here, we make another attempt to describe the finiteness of objects themselves, and the theorems in [2] which were omitted of discussion in [3] will be wholly generalized. Theorem 1, Theorem 2 and Theorem 3 below correspond to Lemma 9, Lemma 10 and Theorem 8 in [2] respectively.

As for the definitions of terms such as *compatible family of morphisms*, *finitary objects* and *ultraproducts*, refer to [3], and for more basic terms of categories, to Isbell [1].

§ **1.** Let $\mathfrak{C}$ be an abstract locally small category which is complete to the both sides, $Ob(\mathfrak{C})$ the collection of all its objects, and for A, $B \in Ob(\mathfrak{C})$, $\mathfrak{C}(A, B)$ the set of all morphisms from A to B.

Definition. For objects G and B, G is said to *separate* B, if for any coterminal morphisms $f, f': B \rightrightarrows B'$ such that $f \neq f'$, there exists an $s: G \to B$ such that $fs \neq f's$. An object B is called *finite*, if there exists a $G \in Ob(\mathfrak{C})$ such that G separates all powers of B and $\mathfrak{C}(G, B)$ consists of only finite number of morphisms. G is said to *represent the finiteness of* B.

Theorem 1. *If an object B is finite, then the diagonal map $d: B \to B^{\Lambda}/D$ to an ultrapower is epimorphic for any set Λ and its ultrafilter D.*

Proof.