26. On Odd Type Galois Extension with Involution of Semi-local Rings*)

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1. Introduction. In [3], the notion of odd type G-Galois extension with involution was defined as follows: If $A\supset B$ is a G-Galois extension and A has an involution $A\to A$; $a\rightsquigarrow\bar{a}$, which is compatible with every element σ of G, i.e. $\sigma(\bar{a})=\overline{\sigma(a)}$ for all $a\in A$, then $A\supset B$ is called a G-Galois extension with involution. A G-Galois extension with involution $A\supset B$ is called odd type, if A has an element u satisfying the following conditions;

1) u is an invertible element in the fixed subring of the center of A by the involution,

2) a hermitian left B-module (A, b_t^u) defined by $b_t^u: A\times A\to B$; $(x,y)\rightsquigarrow t_G(ux\bar{y})=\sum_{\sigma\in G}\sigma(ux\bar{y})$, is isometric to an orthogonal sum of $\langle 1\rangle$ and a metabolic B-module.

If A, B are fields and $A\supset B$ is a G-Galois extension with involution, it was known that $A\supset B$ is odd type if and only if the order of G is odd. In this note, we want to extend this to semi-local rings. When $A\supset B$ is a G-Galois extension with involution of commutative rings, it is easily seen that an odd type G-Galois extension implies $|G|=$odd. For semi-local rings A and B, we shall show that the converse holds in the following cases:

I. The involution is trivial and $|B/\mathfrak{m}|\geqq|G|$ for every maximal ideal m of B, where $|B/\mathfrak{m}|$ and $|G|$ denote numbers of elements of $B/\mathfrak{m}$ and G, respectively.

II. The involution is non-trivial and for each maximal ideal $\mathfrak{m}$ of B the following conditions are satisfied;

1) $|B/\mathfrak{m}|\geqq 2|G|$, 2) if $\bar{\mathfrak{m}}=\mathfrak{m}$, the involution induces a non-trivial one on $A/\mathfrak{m}A$.

III. B is a local ring with maximal ideal $\mathfrak{m}$, and the involution is non-trivial on A but induces a trivial one on $A/\mathfrak{m}A$. Furthermore, $|B/\mathfrak{m}|\geqq|G|$ and $B/\mathfrak{m}$ is either a field with the characteristic not 2 or a finite field. Throughout this paper, every ring is a commutative semi-local ring with identity and $A\supset B$ denotes a G-Galois extension with involution.

2. Galois extension with trivial involution. Lemma 1. *Let*

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