25. *On Decompositions of Linear Mappings among Operator Algebras*

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1. Introduction. Let φ be a $B(H)$-valued function on a set X where $B(H)$ is the algebra of all (bounded linear) operators on a Hilbert space H, and (S) be a property on such φ's. A (closed) subspace M of H *(S)-reduces* φ if M reduces $\varphi(x)$ for all $x \in X$ and $\varphi(x)|M \in (S)$ where $\psi \in (S)$ if ψ has (S). For a subspace N reducing all $\varphi(x)$, the function $\varphi(x)|N$ is *completely non-(S)* if there is no non-zero subspace which (S)-reduces the function.

A strongly closed set P of projections of a von Neumann algebra A is a *Szymanski family* if P satisfies the following conditions (cf. [6]):

(1) *If* $e, f \in P$ *then* $e \wedge f \in P$,

(2) *If* $e, f \in P$ *and* $ef=0$ *then* $e+f \in P$,

(3) *If* $e, f \in P$ *and* $e \geq f$ *then* $e-f \in P$

and

(4) *If* $e \in P$, $f \in \mathrm{proj}\,(A)$ *and* $e \sim f$ (mod. A) *then* $f \in P$. P is called *hereditary* if it satisfies

(5) *If* $e \in P$, $f \in \mathrm{proj}\,(A)$ *and* $e \geq f$ *then* $f \in P$.

If P is a hereditary Szymanski family, then P is a principal ideal of the lattice $L=\mathrm{proj}\,(A)$, cf. [9, Lemma 2], and the largest element e_0 of P is central according to [9, Theorem 5]. Recently Y. Kato and S. Maeda [8] proved that the localization of e_0 in the center of L has a purely lattice theoretic character. Summing up:

Theorem 1. *If P is a Szymanski family in a von Neumann algebra A, then there exists the largest projection e_0 of P in the center of A.*

Let $A=(\varphi(X)\cup\varphi(X)^*)'$ where B' is the commutant of B. A property (S) is called a *Szymanski property* if

$$P=\{e \in \mathrm{proj}\,(A) : \varphi(\cdot)\,|\,eH \in (S)\}$$

is a hereditary Szymanski family. Szymanski [9] proved the following general decomposition theorem for operator valued functions.

Theorem 2. *If (S) is a Szymanski property, then there exists the largest (S)-reducing subspace e_0H such that $\varphi(\cdot)e_0 \in (S)$, and $\varphi(\cdot)e_0^\perp$ is completely non-(S).*

In the present note we shall show that these theorems are applicable to operator algebras. We shall treat the decomposition of expec-