22. On an Asymptotic Property of Spectra of a Random Difference Operator

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We consider a discrete version of the Schrödinger operator with a random potential q:

(1) $$(H^{\omega}u)(a)=-(H^0u)(a)+q(a,\omega)u(a), \qquad a\in Z^{\nu}.$$

Here u is a function on the ν-dimensional lattice space Z^{ν}, H^0 is a second order difference operator defined by

(2) $$(H^0u)(a)=\frac{\sigma^2}{2}\sum_{i=1}^{\nu}\{u(a_1,\cdots,a_i-1,\cdots,a_{\nu})-2u(a)+u(a_1,\cdots,a_i+1,\cdots,a_{\nu})\}, \qquad a\in Z^{\nu},$$

with a positive constant σ and $\{q(a,\omega);a\in Z^{\nu}\}$ is a family of random variables defined on a certain probability space $(\Omega,\mathcal{B},P)$. The only assumption we make on random variables $\{q(a,\omega);a\in Z^{\nu}\}$ is that they form a stationary random field. Their common distribution function will be denoted by $F(x)$:

(3) $$F(x)=P(q(0)\leqq x), \qquad x\in R^1.$$

Denote by $L^2(Z^{\nu})$ the space of all square summable functions on Z^{ν} with inner product $(u,v)=\sum_{a\in Z^{\nu}}u(a)v(a)$. Since H^0 is a bounded symmetric operator on $L^2(Z^{\nu})$, it is easy to see that the operator H^{ω} restricted to the space $C_0(Z^{\nu})$ of functions with finite supports is essentially self-adjoint and that its self-adjoint extension A^{ω} can be described as follows:

(4) $$\begin{cases}\mathcal{D}(A^{\omega})=\{u\in L^2(Z^{\nu});H^{\omega}u\in L^2(Z^{\nu})\}\\ A^{\omega}u=H^{\omega}u \qquad u\in\mathcal{D}(A^{\omega}).\end{cases}$$

Let $\{E_{\lambda}^{\omega};\lambda\in R^1\}$ be the resolution of the identity associated with A^{ω}: $A^{\omega}=\int_{-\infty}^{\infty}\lambda dE_{\lambda}^{\omega}$.

It turns out that $(E_{\lambda}^{\omega}I_0,I_0)$ is measurable in $\omega\in\Omega$ where $I_0(a)=\delta_0(a)$ $a\in Z^{\nu}$. So we can define

(5) $$\rho(\lambda)=\langle(E_{\lambda}^{\omega}I_0,I_0)\rangle$$

where $\langle\ \ \rangle$ is the expectation with respect to the probability measure P. ρ is called the spectral distribution function of the ensemble of operators $\{H^{\omega};\omega\in\Omega\}$. Our present aim is to show that ρ and the distribution function F of $q(0)$ have the same tails asymptotically in the following sense.