21. *On the Boundedness of Integral Transformations with Highly Oscillatory Kernels*

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§ 1. **Preliminaries.** The aim of this note is to prove the $L^2(R^n)$ boundedness of a class of integral transformations which play a fundamental rôle in our notes [2] and [3].

§ 2. **Assumptions.** We shall treat the following integral transformation;

$$Af(x)=\int_{R^n} a(x,y)\exp(i\lambda S(x,y))f(y)dy, \qquad \lambda>0, \tag{1}$$

which is defined at least for $f\in C_0^\infty(R^n)$. Let $|x|$ denote the length of n vector x. Our assumptions are the following;

(A-I) $S(x,y)$ is a real infinitely differentiable function on $R^n\times R^n$.

(A-II) $\Phi=|\mathrm{grad}_x(S(x,y)-S(x,z))|\geqq \Xi_1(x,y,z)\theta(|y-z|)$,
$\Psi=|\mathrm{grad}_y(S(x,y)-S(z,y))|\geqq \Xi_2(x,y,z)\theta(|x-z|)$,

where $\Xi_1(x,y,z)>\delta>0$, $\Xi_2(x,y,z)>\delta>0$, and $\theta(t)=(10\sqrt{n})^{\sigma-1}t$ for $0<t<10\sqrt{n}$ and $=t^\sigma$ for $10\sqrt{n}<t$.

(A-III) For any multi-index α there exists a constant $C>0$ such that we have

$$\left|\left(\frac{\partial}{\partial x}\right)^\alpha (S(x,y)-S(x,z))\right|\leqq C\Phi$$

$$\left|\left(\frac{\partial}{\partial y}\right)^\alpha (S(x,y)-S(z,y))\right|\leqq C\Psi.$$

(A-IV) For any multi-index α there exists a constant $C>0$ such that we have

$$\left|\left(\frac{\partial}{\partial x}\right)^\alpha (a(x,y)a(x,z))\right|\leqq C\Xi_1(x,y,z)^{|\alpha|}$$

$$\left|\left(\frac{\partial}{\partial y}\right)^\alpha (a(x,y)a(z,y))\right|\leqq C\Xi_2(x,y,z)^{|\alpha|}.$$

§ 3. **Result.** Let $\|f\|$ denote the usual L^2 norm of a function f.

Theorem. *If assumptions* (A-I), (A-II), (A-III) *and* (A-IV) *hold, we have estimate*

$$\|Af\|\leqq C\lambda^{-n/2}\|f\|, \qquad \text{for } \lambda>1.$$

Here C is a positive constant independent of λ and f.

§ 4. **Proof.** Let $g_0=0, g_1, g_2, \cdots, g_k, \cdots$ be unit lattice points of R^n. Let $\{\varphi_j(x)\}_{j=0}^\infty$ be a smooth partition of unity in R^n subordinate to the covering of open cubes of side 2 with center at these points. We