20. *Conductor of Elliptic Curves with Complex Multiplication and Elliptic Curves of Prime Conductor*

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1. In Table I, we give the conductor of all the elliptic curves defined over Q, the rational number field, with complex multiplication with the j-invariants in Q. In Table II, we give all the elliptic curves defined over Q of prime conductor $N \leqq 101$, up to isogeny, under Weil's conjecture for $\Gamma_0(N)$.

2. Let E be an elliptic curve over Q with complex multiplication. Then End $(E) \otimes Q = K$ must be an imaginary quadratic field and End (E) is a subring of R, the ring of integers of K, with finite index. Such a subring is of the form $R_f = Z + fR$, where Z is the ring of rational integers and f is the conductor of R_f. Then End (E) has the class number one and there are 13 such R_f's. Hence there are 13 corresponding elliptic curves and the j-invariants of these curves are well-known ([1]), so we can write explicitly their Weierstrass (not always minimal) models. The conductor of these 13 curves can be calculated as Table I below. As is well-known, the reduction at a prime $(\neq 2, 3)$ dividing the conductor N of an elliptic curve with complex multiplication is an additive type, that is to say, $\mathrm{ord}_p N = 2$ if $p \neq 2, 3$, therefore it is sufficient to treat the 2 and 3-factors of N in order to calculate N explicitly. Hence in the last column in Table I, we give only the number $2^{e_2}, 3^{e_3}$, where $N = \prod p^{e_p}$.

Table I

Curve	f	K	model	2,3-factors of N
1	1	$Q[\sqrt{-1}]$	$y^2+x^3+Dx=0$ $\Delta=-2^6D^3$, $j=12^3$ (D: fourth power free)	2^5 if $D\equiv 3$ or $D/4\equiv 1$ 2^6 if $D\equiv 1$ or $D/4\equiv 3$ 2^8 if $2\|D$ or $2^3\|D$
2	1	$Q[\sqrt{-2}]$	$y^2+x^3+4Dx^2+2D^2x=0$ $\Delta=2^9D^6$, $j=20^3$	2^8
3	1	$Q[\sqrt{-3}]$	$y^2+x^3+D=0$ $\Delta=-2^43^3D^2$, $j=0$ (D: sixth power free)	2^23^2 if i) D: cubic, ii) $D\equiv 3$ and iii) $3\nmid D$ or $3^3\|D$ 2^43^2 if i) D: cubic, ii) $D\equiv 1$ and iii) $3\nmid D$ or $3^3\|D$