19. *Theory of Tempered Ultrahyperfunctions. I*

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In this paper we consider the tempered ultrahyperfunction which was introduced by Sebastião e Silva [3] and Hasumi [1] in the name of tempered ultradistribution. We will give some precisions on the work of M. Hasumi. The same idea was developped in [2] for the Fourier ultrahyperfunction.

§1. **The basic spaces $\boldsymbol{H}(\boldsymbol{R}^n; \boldsymbol{O}')$ and $\boldsymbol{H}(\boldsymbol{R}^n; \boldsymbol{K}')$.** Let $K' \subset \boldsymbol{R}^n$ be a convex compact set. Put $h_{K'}(x) = \sup\{\langle x, \xi\rangle; \xi \in K'\}$, $\langle x, \xi\rangle$ being the canonical inner product of $\boldsymbol{R}^n \times \boldsymbol{R}^n$. Remark $h_{K'}(x) = k'|x|, |x| = |x_1| + |x_2| + \cdots + |x_n|$ for $K' = [-k', k']^n$. Let $H_b(\boldsymbol{R}^n; K')$ be the space of all C^∞ functions f on $\boldsymbol{R}^n$ such that $\exp(h_{K'}(x))D^p f(x)$ is bounded in $\boldsymbol{R}^n$ for any multi-index p. D^p denotes the partial differential operator

$$\frac{\partial^{|p|}}{\partial x_1^{p_1}\partial x_2^{p_2}\cdots\partial x_n^{p_n}}, \quad p = (p_1, p_2, \cdots, p_n), \quad |p| = p_1 + p_2 + \cdots + p_n.$$

We define in $H_b(\boldsymbol{R}^n; K')$ seminorms

$$(1) \qquad \|f\|_{K',k} = \sup\{\exp(h_{K'}(x))|D^p f(x)|; |p| \leqslant k, x \in \boldsymbol{R}^n\}$$

for $k = 0, 1, 2, \cdots$. With these seminorms, the space $H_b(\boldsymbol{R}^n; K')$ is a Fréchet space. If K'_1 and K'_2 are two convex compact sets in $\boldsymbol{R}^n$ such that $K'_1 \subset K'_2$, then the canonical injection

$$(2) \qquad H_b(\boldsymbol{R}^n; K'_2) \hookrightarrow H_b(\boldsymbol{R}^n; K'_1)$$

is continuous.

Let O' be a convex open set of $\boldsymbol{R}^n$. We define

$$(3) \qquad H(\boldsymbol{R}^n; O') = \lim_{K' \subset O'} \operatorname{proj} H_b(\boldsymbol{R}^n; K'),$$

where K' runs through the convex compact sets contained in O' and the projective limit is taken following the canonical injections (2). If O'_1 and O'_2 are two convex open sets in $\boldsymbol{R}^n$ such that $O'_1 \subset O'_2$, we have the canonical injection: $H(\boldsymbol{R}^n; O'_2) \hookrightarrow H(\boldsymbol{R}^n; O'_1)$.

For a convex compact set K' of $\boldsymbol{R}^n$, we put

$$(4) \qquad H(\boldsymbol{R}^n; K') = \lim_{K'' \supset K'} \operatorname{ind} H_b(\boldsymbol{R}^n; K''),$$

where K'' runs through the convex compact sets such that K' is contained in the interior of K'' and the inductive limit is taken following the canonical mappings (2). If K'_1 and K'_2 are convex compact sets in $\boldsymbol{R}^n$ such that $K'_1 \subset K'_2$, then we have the canonical injection: $H(\boldsymbol{R}^n; K'_2) \hookrightarrow H(\boldsymbol{R}^n; K'_1)$.

Theorem 1. *Let O' be a convex open set in $\boldsymbol{R}^n$ and K' be a convex*