36. *Groups which Act Freely on Manifolds*

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1. Introduction. This paper is concerned with groups which act freely on closed manifolds.[1] Two theorems will be proved as application of theorems in [6].

For any odd integer r, let $P''(48r)$ denote the group with generators X, Y, Z, A and relations

$$X^2=Y^2=Z^2=(XY)^2, \quad A^{3r}=1,$$
$$ZXZ^{-1}=YX, ZYZ^{-1}=Y^{-1}, \quad AXA^{-1}=Y,$$
$$AYA^{-1}=XY, \quad ZAZ^{-1}=A^{-1}.$$

J. Milnor [5] asks if the group $P''(48r)$ can act freely on the 3-sphere. We shall prove

Theorem 1. *If $r>1$, the group $P''(48r)$ can not act freely on any closed manifold M having the* mod 2 *homology of the* $(8t+3)$*-sphere* $(t\geqq 0)$.

We note that the assertion of Theorem 1 is stated in Corollary 4.17 of [4] whose proof is not correct if r is a power of 3. (See also [6].)

F.B. Fuller [3] proves the following: Let X be a compact polyhedron such that the Euler characteristic is not zero, and let $h: X\to X$ be a homeomorphism. Then the iterate h^i for some $i\geqq 1$ has a fixed point. This shows that if G is a group acting freely on X then any element of G has finite order. By proving a theorem similar to the Fuller theorem, we shall show

Theorem 2. *Let M be a $(2n+1)$-dimensional closed manifold such that the* mod 2 *semichracteristic $\hat{\chi}(M; Z_2)$ is not zero, and let G be a group acting freely on M. Then, for any $T\in G$ of order* 2 *and for any $S\in G$, the commutator $[S, T]$ has finite order.*

2. Proof of Theorem 1. It follows that the subgroup in $P''(48r)$ generated by $\{X, Y\}$ is the quaternion group $Q(8)$ of order 8 and it is a normal subgroup. We see also that the quotient group $P''(48r)/Q(8)$ is generated by the coset $T=[Z]$ and $S=[A]$ with relations $T^2=(TS)^2=S^{3r}=1$, and hence it is the dihedral group $D(6r)$ of order $6r$.

Suppose we have a free action of $P''(48r)$ on M. Let $N=M/Q(8)$ denote the quotient manifold of M under the action of $Q(8)$. Then there is a natural free action of $D(6r)$ on N. Since the homology group

1) In this paper we work in the topological category.