34. *Unipotent Elements and Characters of Finite Chevalley Groups*

By Noriaki KAWANAKA
Department of Mathematics, Osaka University

(Comm. by Kenjiro SHODA, M. J. A., March 12, 1975)

Let $\mathfrak{G}$ be a connected semisimple linear algebraic group defined over an algebraically closed field K of characteristic $p>0$, and σ a surjective endomorphism of $\mathfrak{G}$ such that the group $\mathfrak{G}_\sigma$ of fixed points is finite. A finite group $G=\mathfrak{G}_\sigma$ obtained in this manner is called a finite Chevalley group. The purpose of this note is to announce some results concerning unipotent elements and (complex) characters of a finite Chevalley group $G=\mathfrak{G}_\sigma$. The proof is given in the author's forthcoming paper [8]. After the paper [8] was submitted to Osaka Journal of Mathematics, the author received two preprints [9] and [10], in which Theorems II, IV and V below are proved independently.

1. Let (G,B,N,S) be a Tits system (or BN-pair) associated to a finite Chevalley group G. We denote by W its Weyl group. Let G^1 be the set of unipotent elements (or p-elements) of G, and U the p-Sylow subgroup of G contained in B. For a finite set A, $|A|$ denotes the number of its elements.

Theorem I. *Let w be an arbitrary element of W, and w_S the element of W of maximal length. Then the number of unipotent elements of G contained in the double coset BwB is $|BwB\cap w_SUw_S^{-1}|\,|U|$.*

Corollary. $|G^1|=|U|^2$.

Remarks. (a) In [8], we will prove a formula for the number of unipotent elements contained in $BwB\cap P$, where P is an arbitrary parabolic subgroup of G. Theorem I above is a special case of this formula.

(b) The above corollary is originally proved by R. Steinberg [7].

2. An element x of $\mathfrak{G}$ is called regular if $\dim Z_{\mathfrak{G}}(x)=\operatorname{rank}\mathfrak{G}$, where $Z_{\mathfrak{G}}(x)$ is the centralizer of x in $\mathfrak{G}$. In [6], Steinberg proved the existence of regular unipotent elements of $\mathfrak{G}$. For example, if $\mathfrak{G}=SL_n$, a unipotent element of $\mathfrak{G}$ is regular if and only if its Jordan normal form consists of a single block. Below, we call an element of $G=\mathfrak{G}_\sigma$ regular if it is regular as an element of $\mathfrak{G}$.

Theorem II. *Assume that the characteristic p is good* (*see* [1; *Part E*]) *for $\mathfrak{G}$. Let g be an arbitrary element of $G=\mathfrak{G}_\sigma$, and C a regular unipotent conjugacy class of G. Then the number $|Bg\cap C|$ depends neither on g nor C.*