33. *Local Solvability of a Class of Partial Differential Equations with Multiple Characteristics*

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§ 1. **Introduction.** The present paper is concerned with local solvability for the following type of operators with C^∞ coefficients

$$L(x;\partial_x)=P(x;\partial_x)+Q(x;\partial_x)+R(x;\partial_x) \qquad (x\in \boldsymbol{R}^n),$$

where $P(x;\partial_x)$, $Q(x;\partial_x)$, $R(x;\partial_x)$ are the principal part of order s, the homogeneous part of order $s-1$, and the part of order $s-2$, respectively. When $P(x;\partial_x)$ is of principal type, L. Nirenberg-F. Treves [3] and R. Beals-G. Fefferman [1] established the necessary and sufficient condition for local solvability. On the other hand, when $P(x;\partial_x)$ has double characteristics, a necessary condition is given by F. Cardoso-F. Treves [2]. In that paper they pointed out that the subprincipal part of $L(x;\partial_x)$ plays an important role.

In this paper, we give a sufficient condition under some hypotheses not only for the principal part but for the subprincipal part. A forthcoming article will give a detailed proof. Let V_x be a neighbourhood of the origin in $\boldsymbol{R}^n_x$, and $\boldsymbol{S}^{n-1}_\xi$ be the unit sphere in $\boldsymbol{R}^n_\xi$. For the principal symbol $P(x;\xi)$, we assume that the characteristics of $P(x;\xi)$ have locally constant multiplicities in $V_x\times \boldsymbol{S}^{n-1}_\xi$. Under this assumption when we divide $J=\{(x,\xi)\in V_x\times S^{n-1}_\xi \mid P(x;\xi)=0\}$ into the connected components $\{J_k\}$, $P(x;\xi)$ vanishes of constant order m_k on J_k. Moreover, for simplicity, we assume that $P(x;\partial_x)$ has real coefficients.

§ 2. **Statement of the theorem.** Let us put

$$J^{(2)}=\{(x,\xi)\in J \mid \mathrm{grad}_\xi P(x;\xi)=0\}$$

and divide it into the connected components $\{J^{(2)}_k\}$. For the subprincipal symbol

$$\Pi(x;\xi)=Q(x;\xi)-\frac{1}{2}\sum_{j=1}^{n}\frac{\partial^2}{\partial_{x_j}\partial_{\xi_j}}P(x;\xi),$$

we assume that on each $J^{(2)}_k$, $\Pi(x;\xi)$ satisfies one of the following conditions:

(A) $\mathrm{Re}\,\Pi(x;\xi)\neq 0$ on $J^{(2)}_k$.

(B) $\Pi(x;\xi)\equiv 0$ on $J^{(2)}_k$ and if $m_k\geqslant 3$ moreover $\mathrm{grad}_\xi\,\mathrm{Re}\,\Pi(x;\xi)\neq 0$ on $J^{(2)}_k$.

When the above assumptions are satisfied, we have the following proposition.

Proposition. *For arbitrary real number l, there exists a neigh-*