58. Micro-local Properties of $\prod_{j=1}^{n} f_{j+}^{s_j}$ *)

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In connection with Sato's conjecture in S-matrix theory it has become important to investigate the micro-local properties of the function of the form $\prod_{j=1}^{n} f_{j+}^{s_j}$. See Kawai-Stapp [7] for example. The purpose of this note is to present some basic theorems on the micro-local structure of the function of the above form. The application of the results to the investigation of b-functions will be given somewhere else by the first author. See Kawai-Stapp [7] for the application of the results of this note to the micro-local study of the S-matrix and related functions.

The essential tool in our proof is the desingularization theorem of Hironaka (Hironaka-Lejeune-Teissier [5]). The usefulness of the desingularization theorem in investigating analytic properties of $\prod_{j=1}^{n} f_{j+}^{s_j}$ was first conjectured by Professor I. M. Gel'fand. See Bernstein-Gel'fand [3] and Atiyah [1]. See Björk [4] also. Note that Bernstein [2] proved Theorem 1 without making use of the desingularization theorem in the case when $n=1$ and f_1 is a polynomial.

In this note we use the same notations as in Sato-Kawai-Kashiwara [8] and Kashiwara [6] and do not repeat their definitions.

Theorem 1. *Let f_j $(j=1, \cdots, n)$ be real valued real analytic functions defined on a real analytic manifold M. Let s_j $(j=1, \cdots, n)$ be complex numbers with non-negative real part. Then there exists a maximally overdetermined system $\mathcal{M}$ of linear differential equations such that $u=\prod_{j=1}^{n} f_{j+}^{s_j}$ is a solution of system $\mathcal{M}$.*

Corollary 2. *Under the same assumptions as in Theorem 1 we can find a locally finite family of locally closed submanifolds N_i $(i=1, 2, \cdots)$ of M such that*

(1) $$S.S. \prod_{j=1}^{n} f_{j+}^{s_j} \subset \bigcup_i \sqrt{-1}\, T_{N_i}^{*}M \cup \left(\bigcup_i \sqrt{-1}\, T_{N_i}^{*}N_i\right)$$

holds.

Relation (1) *implies further that*

(2) $$S.S. \prod_{j=1}^{n} f_{j+}^{s_j} \subset \bigcup_i \sqrt{-1}\, S_{N_i}^{*}M.$$

Theorem 3. *Let M be a real analytic manifold and X be its com-*

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