57. *On Involutive Systems of Partial Differential Equations whose Characters of Order more than One Vanish*

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0. Introduction. The structure of involutive systems of partial differential equations with one unknown function of *two* independent variables was recently investigated in detail and Darboux's method of integration was extended to those systems by the author [6], [7]. Our main aim is firstly to investigate the structure of involutive systems of partial differential equations with one unknown function of *several* independent variables whose characters of order more than one vanish and secondly to obtain a method of integration similar to Darboux's method for such systems. Although some of our results are derived from E. Cartan's ones on involutive differential systems whose characters of order more than one vanish (Cartan [3], §§ 97–100), our results are much more complete than E. Cartan's. The descriptions of our results are similar to those in the case of two independent variables obtained by the author himself [7]. However the arguments concerning algebraic considerations are pretty different from those in the memoir [7]. All notions which appear in this note are assumed to be in the category of real or complex analyticity although all arguments except when the existence theorem of Cartan-Kähler is applied can be done in the category of differentiability. Details of this note will be published elsewhere.

1. Involutive systems. Let Φ be a system of partial differential equations of order m with one unknown function. Φ is a system defined in $J^m(M,N,\rho)$, the space of m-jets of sections of a fibered manifold (M,N,ρ) in which $\dim M=\dim N+1$. Let $(x_1,\cdots,x_n,z)$ be a local coordinate system of M ($n=\dim N$) associated with (M,N,ρ). Let $p_{i_1\cdots i_l}(j^m_a(f))$ denote $\partial^l z(x)/\partial x_{i_1}\cdots\partial x_{i_l}(a)$, where $z(x_1,\cdots,x_n)$ denotes z-coordinate of the section f around $a\in N$. A local coordinate system of $J^m(M,N,\rho)$ is given by

$$(x_1,\cdots,x_n,z,p_{i_1\cdots i_l};\ 1\leqq i_1,\cdots,i_l\leqq n,\quad 1\leqq l\leqq m).$$

Let X be a point of $I\Phi$, the set of integral points of Φ. We shall denote $r_m(X)=\dim\langle \pi_m^* dF\,;\,F\in\Phi_X\rangle$, $r_{m+1}(X)=\dim\langle\pi^*_{m+1}dF\,;\,F\in(p\Phi)_{\tilde X}\rangle$, where $\pi^*_m dF$ denotes $\sum_{i_1\leqq\cdots\leqq i_m}\partial F/\partial p_{i_1\cdots i_m}dp_{i_1\cdots i_m}\in T^*_X(J^m)$, $\langle v_\lambda\,;\,\lambda\in\Lambda\rangle$ denotes the vector space spanned by $\{v_\lambda\,;\,\lambda\in\Lambda\}$, $p\Phi$ is the (total) prolongation of Φ