56. *Note on Continuation of Real Analytic Solutions of Partial Differential Equations with Constant Coefficients**)

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(Comm. by Kôsaku YOSIDA, M. J. A., April 12, 1975)

In [1], [2] and [3] we have given some results on continuation of real analytic solutions of linear partial differential equations with constant coefficients to convex sets K of various types. In this note we remark that the assumption of the convexity of K can be much weakened. This problem has been presented by Professor S. Ito. Also I am indebted to Professor H. Komatsu for the improvement of the result. I am very greteful for their valuable suggestions.

Theorem 1. *Let K be a compact set in $\boldsymbol{R}^n$ such that $\boldsymbol{R}^n\setminus K$ is connected. Let $p(D)$ be a $t\times s$ matrix of linear partial differential operators with constant coefficients, and let p' be its transposed matrix. Assume that* $\mathrm{Hom}\,(\mathrm{Coker}\,p', \mathcal{P})=0$ *and that* $\mathrm{Ext}^1\,(\mathrm{Coker}\,p', \mathcal{P})$ *has no elliptic components, where $\mathcal{P}$ denotes the ring of polynomials. Then, for any open neighborhood U of K we have $A_p(U\setminus K)/A_p(U)=0$, namely, every real analytic solution of $p(D)u=0$ can be uniquely continued to U.*

Proof. Take $u\in A_p(U\setminus K)$. By the vanishing of the cohomology group $H^1(V, A)$ for any open set $V\subset \boldsymbol{R}^n$, we can take $f\in[A(\boldsymbol{R}^n\setminus K)]^s$ and $g\in[A(U)]^s$ such that

$$u=f-g \qquad \text{on } U\setminus K.$$

The assumption implies

$$0=p(D)u=p(D)f-p(D)g \qquad \text{on } U\setminus K.$$

Hence $p(D)f$ and $p(D)g$ define an element h of $A_{p_1}(\boldsymbol{R}^n)$, where p_1 is the compatibility system of p. Let $V\supset K$ be a relatively compact convex open set. Then by the existence theorem (see, e.g., [5], Theorem 1) we can find $V\in[A(V)]^s$ such that $p(D)v=h$ on V. Thus we have

$$f-v\,|_{V\setminus \mathrm{ch} K}\in A_p(V\setminus \mathrm{ch}\,K),$$

where $\mathrm{ch}\,K$ denotes the convex hull of K. By Theorem 2.3 of [2], we obtain a unique continuation $[f-v]\in A_p(V)$ of $f-v\,|_{V\setminus \mathrm{ch} K}$. Since $\boldsymbol{R}^n\setminus K$ is connected, $[f-v]$ agrees with $f-v$ whenever both are defined. Therefore

$$[u]=[f-v]+v-g$$

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*) Partially supported by Fûjukai.