53. On a Characterization of L^2-well Posed Mixed Problems for Hyperbolic Equations of Second Order

By Rentaro AGEMI
Department of Mathematics, Hokkaido University

(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1975)

§ 1. **Introduction and results.** Let Ω be a domain in an n-dimensional euclidean space R^n with smooth boundary $\partial\Omega$. Let P be a t-strictly hyperbolic operator of second order defined in the cylinder $R^1\times\bar{\Omega}$ and B a boundary operator of first order defined on $R^1\times\partial\Omega$. Furthermore we assume that the boundary $\partial\Omega$ is non-characteristic for P and B and the coefficients of P and B are smooth and constant outside a compact set of $R^1\times\bar{\Omega}$. We then consider the following mixed problem (P,B):

$$P(t,x;D_t,D_x)u(t,x)=f(t,x) \qquad (t,x)\in R^1\times\Omega \quad t>0,$$
$$B(t,x;D_t,D_x)u(t,x)=g(t,x) \qquad (t,x)\in R^1\times\partial\Omega \quad t>0,$$
$$D_t^j u(0,x)=h_j(x) \qquad (j=0,1) \quad x\in\Omega.$$

Here $D_t=-i(\partial/\partial t)$, $D_k=-i(\partial/\partial x_k)$ $(k=1,\cdots,n)$ and $D_x=(D_1,\cdots,D_n)$.

The aim of this paper is to show the following

Theorem. A mixed problem (P,B) is L^2-well posed if and only if every constant coefficients problem frozen the coefficients at a boundary point is L^2-well posed.

For the L^2-well posedness of mixed problems see [3].

The "only if" part of Theorem is a special case of [2], Theorem 1 which is proved by using the results in [4], [6]. When the coefficients of B are real valued, the author characterized, using the method in [3], L^2-well posed mixed problems with constant coefficients by the inequalities among the coefficients and proved the "if" part of Theorem by energy method ([1]). When the coefficients of B are complex valued, a characterization of L^2-well posed mixed problems with constant coefficients is obtained in the same direction as real case ([8]). In general, a mixed problem is L^2-well posed whenever Lopatinski determinant does not vanish ([5], [10]). Under the assumption of L^2-well posedness, Lopatinski determinant does not vanish in the interior of the most inner normal cone (11]) and also does not vanish for $\operatorname{Im}\tau<0$ where τ is the covariable of t ([4]). When Lopatinski determinant vanishes only on the real points where the roots λ are simple, a mixed problem is L^2-well posed in the case of second order ([2], [9]). Here λ is a root of characteristic polynomial with respect to the covariable of normal direction to $\partial\Omega$. Thus the "if" part of theorem is proved if a