47. *Local Theory of Fuchsian Systems. I*

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1. Introduction. In this paper, we consider a completely integrable system

$$dX=\Big(\sum_{i=1}^{n}\frac{P_i(x)}{x_i}dx_i\Big)X, \tag{1}$$

where $P_i(x)$, $i=1,\cdots,n$ is an $m\times m$ matrix holomorphic at $x=0$, say

$$P_i(x)=\sum_{k\geq 0}P_{i,k}x^k. \tag{2}$$

Here k denotes a multi-index $(k_1,\cdots,k_n)$, k_i a nonnegative integer, $0=(0,\cdots,0)$, and $x^k=x_1^{k_1},\cdots,x_n^{k_n}$. For two multi-indices k and l, "$k\geq l$" means "$k_i\geq l_i$ for all i" and "$k>l$" means "$k\geq l$ and $k_i>l_i$ for some i". We propose to find out the dominant coefficients in $\{P_{i,k}\}$ which determine the local behavior of the solution of (1).

A change of variables $X=U(x)Y$ with $U(x)$ invertible holomorphic at $x=0$, transforms (1) into the system of the form

$$dY=\Big(\sum_{i=1}^{n}\frac{Q_i(x)}{x_i}dx_i\Big)Y \tag{3}$$

with

$$\sum_{i=1}^{n}\frac{Q_i(x)}{x_i}dx_i=U(x)^{-1}\Big(\sum_{i=1}^{n}\frac{P_i(x)}{x_i}dx_i\Big)U(x)-U(x)^{-1}dU(x). \tag{4}$$

First, we determine $U(x)$ in such a way that (3) has a '*reduced*' form, of which the definition is given in Section 4. Next, we show that by a suitable substitution $Y=x_1^{L_1}\cdots x_n^{L_n}Z$ with $L_i=\operatorname{diag}(l_i^1,\cdots,l_i^m)$, where l_i^α is a nonnegative integer, equation (3), which has a '*reduced*' form, can be changed to the equation $dZ=(\sum_{i=1}^n(B_i/x_i)dx_i)Z$ with constant matrices $B_1,\cdots,B_n$.

When preparing this note, we were communicated from T. Kimura, that R. Gérard was solving a problem analogous to ours.

2. Convergence theorem. We prepare a convergence theorem which will be used later.

Theorem 1. *Let*

$$du=\Big(\sum_{i=1}^{n}\frac{F_i(x)}{x_i}dx_i\Big)u \tag{5}$$

be a completely integrable system, where u is a vector and

$$F_i(x)=\sum_{k\geq 0}F_{i,k}x^k,\qquad i=1,2,\cdots,n$$

are matrices convergent and holomorphic for $|x|<\varepsilon$. Then any formal