46. *Theory of Tempered Ultrahyperfunctions. II*

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We continue our study of tempered ultrahyperfunctions and use the same notations as in our previous note [5]. In this paper, we consider exclusively the 1-dimensional case.

§ 1. Fourier transformation of distributions with properly convex support. Let $K'=[a, b]$ be a closed interval in $\boldsymbol{R}$. We put

(1) $$h_{K'}(x)=\sup\{x\xi\,;\,\xi\in[a,b]\}=\begin{cases}bx & \text{for } x\geqslant 0,\\ ax & \text{for } x<0.\end{cases}$$

We denote by $H(\boldsymbol{R}\,;K')$ the space of all C^∞ functions f on $\boldsymbol{R}$ for which there exists a constant $\varepsilon>0$ such that for any integer $p\geqslant 0$, $\exp(h_{K'}(x)+\varepsilon|x|)D^pf(x)$ is bounded in $\boldsymbol{R}$, where $D^p=d^p/dx^p$. $H(\boldsymbol{R}\,;K')$ is the inductive limit of FS spaces. The dual space $H'(\boldsymbol{R}\,;K')$ of $H(\boldsymbol{R}\,;K')$ is a space of distributions of exponential growth ([5]).

Proposition 1. *Let β be a C^∞ function on $\boldsymbol{R}$ such that $0\leqslant\beta(x)\leqslant 1$, $\beta(x)=1$ for $x\geqslant B$ (resp. $x\leqslant -B$) and $\beta(x)=0$ for $x\leqslant -B$ (resp. $x\geqslant B$), with some constant $B>0$. Then $\beta(x)\exp(-ix\zeta)\in H(\boldsymbol{R}\,;K')$ if and only if $\operatorname{Im}\zeta<-b$ (resp. $\operatorname{Im}\zeta>-a$).*

Proof. Remark first

(2) $$|e^{-ix\zeta}|=e^{x\eta},\ |D^pe^{-ix\zeta}|=|\zeta^p|\,e^{x\eta},\qquad \text{where } \zeta=\xi+i\eta.$$

Therefore, we have

$$\exp(h_{K'}(x)+\varepsilon|x|)\,|D^pe^{-ix\zeta}|=\begin{cases}|\zeta^p|\exp(b+\varepsilon+\eta)x & \text{for } x>0,\\ |\zeta^p|\exp(a-\varepsilon+\eta)x & \text{for } x<0,\end{cases}$$

from which follows the proposition. q.e.d.

We put

(3) $$\begin{aligned} H'_{(+)}(\boldsymbol{R}\,;K')&=\{T\in H'(\boldsymbol{R}\,;K')\,;\,\operatorname{supp}T\subset[-A,\infty)\text{ for some }A\geqslant 0\},\\ H'_{(-)}(\boldsymbol{R}\,;K')&=\{T\in H'(\boldsymbol{R}\,;K')\,;\,\operatorname{supp}T\subset(-\infty,A]\text{ for some }A\geqslant 0\},\\ H'_{(0)}(\boldsymbol{R}\,;K')&=\{T\in H'(\boldsymbol{R}\,;K')\,;\,\operatorname{supp}T\subset[-A,A]\text{ for some }A\geqslant 0\}.\end{aligned}$$

These are linear subspaces of $H'(\boldsymbol{R}\,;K')$. We put further

(3′) $$\begin{aligned} H'_{+}(\boldsymbol{R}\,;K')&=\{T\in H'(\boldsymbol{R}\,;K')\,;\,\operatorname{supp}T\subset[0,\infty)\},\\ H'_{-}(\boldsymbol{R}\,;K')&=\{T\in H'(\boldsymbol{R}\,;K')\,;\,\operatorname{supp}T\subset(-\infty,0]\},\\ H'_{0}(\boldsymbol{R}\,;K')&=\{T\in H'(\boldsymbol{R}\,;K')\,;\,\operatorname{supp}T=\{0\}\}.\end{aligned}$$

The spaces $H'_{+}(\boldsymbol{R}\,;K')$, $H'_{-}(\boldsymbol{R}\,;K')$ and $H'_{0}(\boldsymbol{R}\,;K')$ are closed subspaces of the space $H'(\boldsymbol{R}\,;K')$.

Let $T\in H'_{(+)}(\boldsymbol{R}\,;K')$ and $\operatorname{supp}T\subset[-A,\infty)$ (resp. $T\in H'_{(-)}(\boldsymbol{R}\,;K')$ and $\operatorname{supp}T\subset(-\infty,A]$). We choose a C^∞ function β such that $0\leqslant\beta(x)\leqslant 1$, $\beta(x)=1$ for $x\geqslant -A-\delta$ (resp. $x\leqslant A+\delta$) and $\beta(x)=0$ for $x\leqslant -A-2\delta$