68. A Note on Isolated Singularity. I

By Isao NARUKI
Research Institute for Mathematical Sciences, Kyoto University

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0. Introduction. This note attempts to generalize the author's earlier result [6] to higher codimensional case, seeking for more profound base of the study. The remarkable feature is the introduction of the condition (L) which provides a reasonable class of isolated singularities including that of complete intersections; in fact almost all important properties are consequences from this condition.

1. Condition L. Let (X, x) be an isolated singularity, namely, a pair of (complex) analytic space X and a point $x \in X$ such that $X \backslash x$ is non-singular.

Definition. We say (X, x) satisfies the condition (L) if and only if $\mathcal{H}_x^q(\Omega_X^p)=0$ for (p, q) such that $p+q<\dim X$, where Ω_X^p denote the sheaves of analytic p-forms on X for $p=0,1,2,\cdots$.

Let f be an analytic function on X such that $f(x)=0$, $df_z \neq 0$ for any $z \in X\backslash x$. Then $(f^{-1}(0), x)$ is a new isolated singularity, which we shall denote by (Y, y) in the following. (Note $y=x$.) We set as in Brieskorn [2]

$$\Omega_f^p=\Omega_X^p / df \wedge \Omega_X^{p-1}.$$

Now we have

Theorem 1. *Let $n=\dim Y \geqq 2$. Then (X, x) satisfies* (L) *if and only if (Y, y) satisfies* (L) *and* $\dim \mathcal{H}_y^0(\Omega_Y^n)=\dim \mathcal{H}_y^1(\Omega_Y^n)$.

Remark. Even in case $n=1$ the condition (L) for (X, x) implies the condition (L) for (Y, y).

For the proof of Theorem 1 we have introduced the following new condition

(L′) $\qquad \mathcal{H}_x^q(\Omega_f^p)=0 \qquad$ for (p, q) such that $p+q<\dim X$

showing that this is equivalent to the both statements of the theorem whose equivalence is to be proved.

By Hamm [4] we obtain

Corollary 1. *It (X, x) is a complete intersection of hypersurfaces, then it satisfies* (L).

Consider now the spectral sequence ${}'E_2^{p,q}=\mathcal{H}_x^p(\mathcal{H}^q(\Omega_X^{\cdot}))$. These E_2-terms are 0 except ${}'E_2^{p,0}=\mathcal{H}_x^p(\boldsymbol{C})$, ${}'E_2^{0,q}=H^q(\Omega_{X,x}^{\cdot})$, $q>0$. But it can be shown by Bloom-Herrera [1] that $H^{r-1}(\Omega_{X,x}^{\cdot})={}'E_r^{0,r-1} \xrightarrow{d_r} {}'E_r^{r,0}=\mathcal{H}_x^r(\boldsymbol{C})$ is zero map for every $r>0$. Comparing this with another spectral