65. On Some Evolution Equations of Subdifferential Operators

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1. Introduction. In this paper we are concerned with nonlinear evolution equations of a form

$$\frac{du}{dt}+\partial\psi^t u(t)+A(t)u(t)\ni f(t), \qquad 0\leqq t\leqq T \tag{1.1}$$

in a real Hilbert space H. Here for each fixed t, $\partial\psi^t$ is subdifferential of a lower semicontinuous convex function ψ^t from H into $(-\infty,\infty]$, $\psi^t \not\equiv \infty$ and $A(t)$ is a monotone, single valued and hemicontinuous operator which is perturbation in a sense. The effective domain of ψ^t defined by $\{u\in H: \psi^t(u)<+\infty\}=D$ is independent of t. We denote the inner product and the norm in H by $(\ ,\ )$ and $\|\ \ \|$ respectively. Let T be a positive constant.

We assume the following conditions for ψ^t and $A(t)$.

A–(1). For every $r>0$ there exists a positive constant $L_1(r)$ such that

$$|\psi^t(u)-\psi^s(u)|\leqq L_1(r)\,|h(t)-h(s)|\,\{\psi^t(u)+1\}$$

hold if $0\leqq s,\ t\leqq T$, $u\in D$ and $\|u\|\leqq r$, where $h(t)$ is a continuous function with bounded total variation.

A–(2). If $u(t)\in D$ is absolutely continuous on $[a,b]$ $(0\leqq a<b\leqq T)$ then $A(t)u(t)$ is strongly measurable on $[a,b]$ and for any fixed $t_0\in[a,b]$ $A(t_0)u(t)$ is also strongly measurable on $[a,b]$. For any fixed $u\in D$, $A(t)u$ is continuous on $[0,T]$.

A–(3). There are Riemann integrable functions $W_r(t)^2$ on $[0,T]$ and a constant $0<K_r<1/2$ such that

$$\|A(t)u\|\leqq K_r\,\||\partial\psi^t u|\|+W_r(t) \qquad \text{for any } \|u\|\leqq r.$$

A–(4). If $u(t)$ is absolutely continuous and $|\psi^t(u)|+\|u(t)\|\leqq r$, then $A(t)u(t)\leq W_r(t)^2$.

Under the above assumptions we consider the uniqueness and existence of the solution of (1–1) where the solution is defined as follows:

Definition 1.1. We say that $u(t)$ is a solution of (1–1) if and only if $u(t)$ is continuous on $[0,T]$ and absolutely continuous on $(0,T]$ and if (1–1) holds almost everywhere on $[0,T]$.

Theorem 1.1. *Suppose that the assumptions stated above are satisfied. Then we hold the unique solution of* (1–1) *where* $f\in L_2[0,T;H]$ *and the initial date* $u_0\in\bar{D}$.